① $\left(\frac{3}{2-\frac{7}{5}}+\frac{\frac{7}{3}-1}{4}\right):\frac{4}{9}=\left(\frac{3}{\frac{10-7}{5}}+\frac{\frac{7-3}{3}}{4}\right)\cdot\frac{9}{4}=$

$=\left(\frac{3}{\frac{3}{5}}+\frac{\frac{4}{3}}{4}\right)\cdot\frac{9}{4}=\left(3:\frac{3}{5}+\frac{4}{3}:4\right)\cdot\frac{9}{4}=\left(3\cdot\frac{5}{3}+\frac{4}{3}\cdot\frac{1}{4}\right)\cdot\frac{9}{4}=$

$\left(5+\frac{1}{3}\right)\cdot\frac{9}{4}=\frac{15+1}{3}\cdot\frac{9}{4}=\frac{16^{4}}{3}\cdot\frac{9^{3}}{4}=12$ (A.)

② $\sqrt{\frac{9}{25}-\frac{36}{35}+\frac{36}{49}}=\sqrt{\left(\frac{3}{5}\right)^2-\frac{2\cdot3\cdot6}{5\cdot7}+\left(\frac{6}{7}\right)^2}=$

$=\sqrt{\left(\frac{3}{5}\right)^2-2\cdot\frac{3}{5}\cdot\frac{6}{7}+\left(\frac{6}{7}\right)^2}=\sqrt{\left(\frac{3}{5}-\frac{6}{7}\right)^2}=\sqrt{\left(\frac{6}{7}-\frac{3}{5}\right)^2}=$

$=\sqrt{\left(\frac{30-21}{35}\right)^2}=\frac{9}{35}$ (B.)

③ $\overline{x5y}$ soni $(45=5\cdot9)$ ga bo'linishi uchun ham 5 ga, ham 9 ga qoldiqsiz bo'linishi kerak. 5 ga bo'linishi uchun oxiri 5 yoki 0 bilan tugashi kerak. 9 ga bo'linishi uchun raqamlari yig'indisi 9 ga bo'linishi kerak.

$\overline{x50}$

$x+5+0$

$4+5+0=9$

$x=4$

$\overline{x55}$

$x+5+5$

$x+10=18$

$x=8$

$x\to 4$ va 8 ni qabul qilar ekan.

$4+8=12$ (C.)

4. $4^{n+2} \cdot 6^{4-n}$ → Tub kopaytuvchilarga maydalash kerak.

$(2^2)^{n+2} \cdot 2^{4-n} \cdot 3^{4-n} \Rightarrow 2^{2n+4} \cdot 2^{4-n} \cdot 3^{4-n} \Rightarrow 2^{2n+4+4-n} \cdot 3^{4-n} \Rightarrow$

$\Rightarrow 2^{n+8} \cdot 3^{4-n} \Rightarrow$ NBS $\Rightarrow (n+8+1) \cdot (4-n+1) = 24$

$(n+9) \cdot (5-n) = 24 \Rightarrow 5n - n^2 + 45 - 9n = 24 \Rightarrow -n^2 - 4n + 21 = 0 \ /\cdot(-1)$

$n^2 + 4n - 21 = 0 \Rightarrow n_1 + n_2 = -4 \quad n_1 = -7 \quad n_2 = 3$ ✓

$n_1 \cdot n_2 = -21$

D.

5. $f(x) = x^2 + 2ax \quad f'(1) = 0 \quad a = ?$

$f'(x) = 2x + 2a \quad f'(1) = 2 \cdot 1 + 2a = 0 \Rightarrow 2a = -2 \quad a = -2:2$

$a = -1$ D.

6. $n = 2025 \cdot m + 2024 \quad \frac{2025 \cdot m + 2024}{5} \Rightarrow \frac{2020 + 4}{5}$

4 D.

7. $2025x^{2024} - 2024x^{1012} + 2023 = 0 \quad x^{1012} = t$

$2025t^2 - 2024t + 2023 = 0$

$D = 2024^2 - 4 \cdot 2025 \cdot 2023 = 4096576 - 16386300 =$

$= -12289724$

$\emptyset$ A.

(8.) $4a+3b-c=43$ $\quad b+5c=17$ $\quad a, b, c$ turli ragamlar.

1) $b=2 \quad c=3$ $\quad 2+5\cdot 3=17$ ✓

$4a+3\cdot 2-3=43$

$4a+3=43 \Rightarrow a=10$ —

2) $b=7 \quad c=2$ $\quad 7+5\cdot 2=17$ ✓

$4a+3\cdot 7-2=43$

$4a+19=43 \Rightarrow 4a=24 \quad a=6$ ✓

$\overline{abc}+\overline{bca}+\overline{cab}=672+726+267=1665$ (B.)

(9.) $\log_4 5 \cdot \log_5 6 \cdot \log_6 64 = \frac{\log_6 5}{\log_6 4} \cdot \frac{1}{\log_6 5} \cdot \log_6 4^3 = \frac{1}{\log_6 4} \cdot 3\log_6 4 = 3$ (C.)

(10.) $2024^{2025}+2025^{2024}=?$ $\quad 4^{\text{Toq}}=\ldots 4 \quad 5^n=\ldots 5$

$2024^{2025}+2025^{2024}=\ldots 4+\ldots 5=\ldots 9$ (A.)

(11.) $\sqrt{224}=\sqrt{16\cdot 14}=\sqrt{16}\cdot\sqrt{14}=4\sqrt{14}$ $\quad \sqrt{14}$ (B)

(12.) $3^x=100 \quad 3^4=81 \quad 3^5=243$ Demak: $4<x<5$

$|x-3|+|x-6|=x-3-(x-6)=x-3-x+6=3$

Modul ichi musbat bólsa ózidek ochiladi.
Modul ichi manfiy bólsa oldiga minus qóyib ochiladi.
Birinchi modul ichi musbat: ózidek ochildi.
Ikkinchi modul ichi manfiy: oldiga minus qóyib ochildi.
(B.)

(13.) $\sqrt{5}-\dfrac{1}{\sqrt{5}+\dfrac{2}{\sqrt{5}-\dfrac{3}{\sqrt{5}}}}=\sqrt{5}-\dfrac{1}{\sqrt{5}+\dfrac{2}{\dfrac{\sqrt{5}\cdot\sqrt{5}-3}{\sqrt{5}}}}=\sqrt{5}-\dfrac{1}{\sqrt{5}+\dfrac{2\sqrt{5}}{5-3}}=$

$=\sqrt{5}-\dfrac{1}{\sqrt{5}+\dfrac{2\sqrt{5}}{2}}=\sqrt{5}-\dfrac{1}{\sqrt{5}+\sqrt{5}}=\sqrt{5}-\dfrac{1}{2\sqrt{5}}=\dfrac{2\sqrt{5}\cdot\sqrt{5}-1}{2\sqrt{5}}=$

$=\dfrac{2\cdot5-1}{2\sqrt{5}}=\dfrac{9}{2\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{9\sqrt{5}}{2\cdot5}=\dfrac{9\sqrt{5}}{10}$ (C.)

(14.) $\dfrac{30}{6+\sqrt[3]{36}+\sqrt[3]{6}}=\dfrac{30}{\sqrt[3]{6^3}+\sqrt[3]{6^2}+\sqrt[3]{6}}=\dfrac{30}{\sqrt[3]{6}\left(\sqrt[3]{6^2}+\sqrt[3]{6}+1\right)}=$

$=\dfrac{30\cdot\sqrt[3]{6^2}}{\sqrt[3]{6^2}\cdot\sqrt[3]{6}\left(\sqrt[3]{6^2}+\sqrt[3]{6}+1\right)}=\dfrac{\overset{5}{\cancel{30}}\sqrt[3]{6^2}}{\cancel{6}\left(\sqrt[3]{6^2}+\sqrt[3]{6}+1\right)}=\dfrac{5\sqrt[3]{6^2}}{\sqrt[3]{6^2}+\sqrt[3]{6}+1}=$

$=\dfrac{5\cdot\left(\sqrt[3]{6}-1\right)\cdot\sqrt[3]{6^2}}{\left(\sqrt[3]{6}-1\right)\left(\sqrt[3]{6^2}+\sqrt[3]{6}+1\right)}=\dfrac{5\left(\sqrt[3]{6}\cdot\sqrt[3]{6^2}-\sqrt[3]{6^2}\right)}{\sqrt[3]{6^3}-1^3}=\dfrac{\cancel{5}\left(6-\sqrt[3]{36}\right)}{\cancel{5}}=$

$=6-\sqrt[3]{36}$ (D.)

(15.) $x=\dfrac{\pi}{22}\ /\cdot 11 \Rightarrow 11x=\dfrac{\pi}{2}$

$\dfrac{\sin4x\cdot\cos3x+\cos4x\cdot\sin3x}{\cos5x\cdot\cos x+\sin5x\cdot\sin x}=\dfrac{\sin(4x+3x)}{\cos(5x-x)}=\dfrac{\sin7x}{\cos4x}=\dfrac{\sin(11x-4x)}{\cos4x}$

$=\dfrac{\sin\left(\frac{\pi}{2}-4x\right)}{\cos4x}=\dfrac{\cos4x}{\cos4x}=1$ (C.)

(16.) $(x^2-9x+20)\lg(10-x^2)=0$.

$x^2-9x+20=0$ $\quad \lg(10-x^2)=0$

$x_1+x_2=9 \quad x_1\cdot x_2=20$ $\quad \lg_{10}(10-x^2)=0$

$x_1=5 \quad x_2=4$ $\quad 10-x^2=10^0 \Rightarrow 10-x^2=1 \Rightarrow -x^2=1-10$

$x^2=9 \quad x=\pm 3$

A.S. $10-x^2>0 \;/\cdot(-1) \Rightarrow x^2-10<0$ $\quad (-\sqrt{10};\sqrt{10}) \approx (-3,2;3,2)$

5 va 4 lar A.S. ga tushmaydi.

Demak yechim ± 3 $\quad 3\cdot(-3)=-9$ (A.)

(17.) $x^2+\frac{1}{x^2}=34 \Rightarrow \left(x+\frac{1}{x}\right)^2-2\cdot x\cdot\frac{1}{x}=34 \Rightarrow \left(x+\frac{1}{x}\right)^2-2=34$

$\left(x+\frac{1}{x}\right)^2=34+2 \Rightarrow \left(x+\frac{1}{x}\right)^2=36 \Rightarrow x+\frac{1}{x}=\pm 6$

$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2-x\cdot\frac{1}{x}+\frac{1}{x^2}\right)=6\cdot\left(x^2+\frac{1}{x^2}-1\right)=6\cdot(34-1)=$

$=6\cdot 33=198$ (B.)

(18) $x^2+y^2-2x-2\sqrt{2}y+3=0 \Rightarrow x^2-2x+y^2-2\sqrt{2}y+2+1=0$

$x^2-2x+1+y^2-2\sqrt{2}y+\sqrt{2}^2=0 \Rightarrow (x-1)^2+(y-\sqrt{2})^2=0$

$x-1=0 \quad y-\sqrt{2}=0 \Rightarrow x=1 \quad y=\sqrt{2}$

$x+y^2=1+\sqrt{2}^2=1+2=3.$ (D.)

19. $a_n = 4n+1$ $\quad S_{10} = ?$

$n=1 \Rightarrow a_1 = 4\cdot 1+1 = 5$

$n=2 \Rightarrow a_2 = 4\cdot 2+1 = 9$

$d = a_2 - a_1 = 9-5 \Rightarrow \boxed{d=4}$

$$S_{10} = \frac{2a_1+9d}{2}\cdot 10 = \frac{2\cdot 5+9\cdot 4}{2}\cdot \frac{5}{10} =$$

$$= (10+36)\cdot 5 = 46\cdot 5 = 230$$

(D.)

20. $x^2+(a+2)x+b-4=0$ $\quad x_1+x_2 = -(a+2)$ $\quad x_1\cdot x_2 = b-4$

$x_1 = -2$ $\quad x_2 = 3$

$-2+3 = -a-2$

$-2+3+2 = -a \Rightarrow 3 = -a \Rightarrow \boxed{a=-3}$

$(x+4)^2 + a(x+4) + 2b - 6 = 0 \Rightarrow x^2+8x+16+ax+4a+2b-6=0$

$x^2+(8+a)x+10+4a+2b=0$ $\quad x_3+x_4 = -(8+a) = -(8+(-3)) =$

$= -8+3 = -5$ (C.)

21. $9+99+999+9999+\ldots+\underbrace{9999\ldots9}_{x\text{ ta}} = ?$

$10^1-1+10^2-1+10^3-1+10^4-1+\ldots+10^x-1 = \underbrace{10^1+10^2+10^3+10^4+\ldots+10^x}_{\text{geom. progres.}} \underbrace{-1-1-1-1\ldots-1}_{x\text{ ta}}$

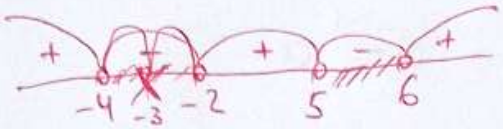
$b_1 = 10$ $\quad q = 10$

$$= \frac{b_1\cdot(q^x-1)}{q-1} - x\cdot 1 = \frac{10\cdot(10^x-1)}{10-1} - x = \frac{10^{x+1}-10}{9} - x = \frac{10^{x+1}-10-9x}{9} =$$

$$= \frac{10^{x+1}-9x-10}{9}$$

(A.)

(22.) $\frac{(x+3)^2 \cdot (x-5)\cdot(x+2)}{(x+4)(x-6)} < 0 \Rightarrow \frac{(x-5)(x+2)}{(x+4)(x-6)} < 0$ $\quad x+3 \neq 0$, $x \neq -3$



$(-4;-3) \cup (-3;-2) \cup (5;6)$ (B.)

(23.) $b_5 = 2 \quad b_8 = 16 \quad b_n = ? \quad b_n = b_1 \cdot q^{n-1} = ?$

$\begin{cases} b_1 \cdot q^7 = 16 \\ b_1 \cdot q^4 = 2 \end{cases} \Rightarrow q^3 = 8 \Rightarrow q^3 = 2^3 \Rightarrow \boxed{q = 2}$

$b_1 \cdot 2^4 = 2 \Rightarrow b_1 = \frac{2}{2^4} = \frac{1}{2^3}$

$b_n = \frac{1}{2^3} \cdot 2^{n-1} = 2^{-3} \cdot 2^{n-1} = 2^{-3+n-1} \Rightarrow b_n = 2^{n-4}$ (C.)

(24.) $\frac{64}{31} + \frac{71}{32} - \frac{27}{33} = a \qquad \frac{2}{31} + \frac{7}{32} + \frac{6}{33} =$

$= \frac{64-62}{31} + \frac{71-64}{32} + \frac{33-27}{33} = \frac{64}{31} - \frac{62}{31} + \frac{71}{32} - \frac{64}{32} + \frac{33}{33} - \frac{27}{33}$

$= \frac{64}{31} - 2 + \frac{71}{32} - 2 + 1 - \frac{27}{33} = \frac{64}{31} + \frac{71}{32} - \frac{27}{33} - 3 =$

$= a - 3$ (D.)

(25.) $40 + x = 14 + x + 9 + x + 7 + x$

$40 - 14 - 9 - 7 = 3x - x$

$10 = 2x$

$x = 5$ (A.)

(26.) $\frac{a}{b} = \frac{c}{d} = \frac{2}{5}$ $\quad \frac{3a+2c}{6b+xd} = \frac{1}{5}$

$\frac{c}{d} = \frac{2}{5} \Rightarrow \boxed{c = \frac{2d}{5}}$

$\frac{a}{b} = \frac{c}{d} \Rightarrow b = \frac{ad}{c}$ $\quad \frac{a}{b} = \frac{2}{5} \Rightarrow a = \frac{2b}{5}$

(26.) $\frac{a}{b} = \frac{c}{d} = \frac{2}{5}$ $\quad \frac{3a+2c}{6b+xd} = \frac{1}{5}$

$\frac{a}{b} = \frac{2}{5} \Rightarrow \boxed{a = \frac{2b}{5}}$ $\quad \frac{c}{d} = \frac{2}{5} \Rightarrow \boxed{c = \frac{2d}{5}}$

$\frac{3 \cdot \frac{2b}{5} + 2 \cdot \frac{2d}{5}}{6b + xd} = \frac{1}{5} \Rightarrow \left(\frac{6b}{5} + \frac{4d}{5}\right) \cdot 5 = 6b + xd$

$6b + 4d = 6b + xd \Rightarrow 4d = xd \Rightarrow x = 4$ (C.)

(27.) $2\sin^2 x - 5\sin x + 2 = 0 \mid :2 \Rightarrow \sin^2 x - 5\sin x + 4 = 0$ $\quad \boxed{\sin x = t}$

$t^2 - 5t + 4 = 0$ $\quad t_1 + t_2 = 5$ $\quad t_1 \cdot t_2 = 4$ $\quad t_1 = 4$ $\quad t_2 = 1$

$t_1 = 2$ $\quad t_2 = \frac{1}{2}$

$\sin x \neq 2$ $\quad \emptyset$ $\quad \sin x = \frac{1}{2}$

$x = (-1)^k \arcsin \frac{1}{2} + \pi k \Rightarrow x = (-1)^k \frac{\pi}{6} + \pi k$

(B.)

(28) $0 < x < \frac{\pi}{2}$ $\quad \frac{1}{\cos x} = \frac{3}{\sin x} \Rightarrow \frac{\sin x}{\cos x} = 3 \Rightarrow tgx = 3$

$1 + tg^2 x = \frac{1}{\cos^2 x} \Rightarrow 1 + 3^2 = \frac{1}{\cos^2 x} \Rightarrow 10 = \frac{1}{\cos^2 x}$

$(10)^{-1} = \left(\frac{1}{\cos^2 x}\right)^{-1} \Rightarrow \frac{1}{10} = \cos^2 x \Rightarrow \cos x = \frac{1}{\sqrt{10}}$ (A.)

(29.) $f(x) = \ln(\lg x)$ $\quad f'(10) = ?$ $\quad f'(x) = \left(\ln(\lg x)\right)' = \frac{1}{\lg x} \cdot (\lg x)' =$

$= \frac{1}{\lg x} \cdot \frac{1}{x \ln 10}$ $\quad f'(10) = \frac{1}{\lg 10} \cdot \frac{1}{10 \ln 10} = \frac{1}{10 \cdot \ln 10}$ (B.)

(30.) $\log_{\sqrt{5}}\left(2 + \log_3(2x+1)\right) = 1 \Rightarrow 2 + \log_3(2x+1) = 5^1 \Rightarrow$

$\Rightarrow \log_3(2x+1) = 3 \Rightarrow 2x + 1 = 3^3 \Rightarrow 2x = 27 - 1$

$2x = 26$

$x = 13$ (C.)