TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

FAN: OLIY MATEMATIKA

Ikkinchi tartibli sirtlar: silindrik sirtlar, sfera, ellipsoid, giperboloid va paraboloidlar, konus.

"Oliy matematika" kafedrasi dotsenti
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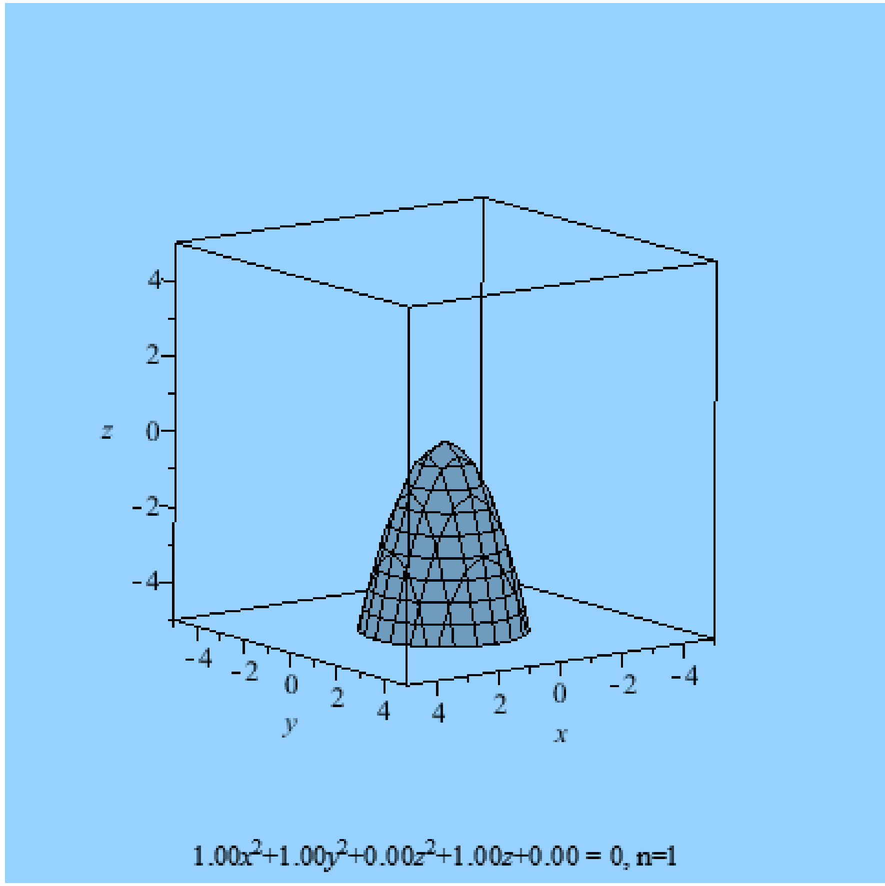
Reja:

1. Ikkinchi tartibli sirtlar
2. Sfera
3. Ellipsoid
4. Giperboloid
5. Paraboloid
6. Silindrik sirtlar
7. Ikkinchi tartibli konus

Silindrik sirtlar

Uch o‘lchovli $Oxyz$ Dekart koordinatalar sistemasida har qanday sirt biror $F(x,y,z)=0$ tenglama bilan yoziladi, bu erda $(x,y,z)-$ sirt ixtiyoriy nuqtasining koordinatasi. Agar $F(x,y,z)\ -\ x,y,z$ o‘zgaruvchilarga nisbatan ikkinchi darajali ko‘phad bo‘lsa, u holda tenglama $F(x,y,z)=0$ **ikkinchi tartibli tenglama** deyiladi, shu tenglama yordamida tasvirlanadigan sirt esa ***ikkinchi tartibli sirt*** deyiladi. Agar sirtning koordinatalar sistemasiga nisbatan joylashishi alohida xususiyatga ega bo‘lsa (masalan, ba’zi koordinatalar sistemalariga nisbatan simmetrik joylashgan bo‘lsa), u holda uning tenglamasi juda sodda ko‘rinishga ega bo‘ladi va u ***kanonik tenglama*** deyiladi.



Silindrik sirtlar

Ikkinchi tartibli sirtning umumiy tenglamasi

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz +$$

$$+2a_1x + 2a_2y + 2a_3z + a_0 = 0$$

ko‘rinishda bo‘ladi,

bu erda $a_{11}, a_{22}, a_{33}, a_{12}, a_{13}, a_{23}, a_1, a_2, a_3, a_0$ – haqiqiy sonlar,

bunda $a_{11}, a_{22}, a_{33}, a_{12}, a_{13}, a_{23}$ – koeffitsientlar bir vaqtda nolga teng emas.

Silindrik sirtlar

Ikkinchi tartibli sirtlar nazariyasida sirtlar klassifikatsiya qilinadi va ularning turli ko‘rinishlari o‘rganiladi. Sirtlarni o‘rganishning usullaridan biri **kesim usulidir**. Bunda sirtlarning koordinata tekisliklariga parallel bo‘lgan yoki koordinata tekisliklarining o‘zi yordamidagi kesimlari o‘rganiladi. Hosil bo‘lgan kesimlarning ko‘rinishiga qarab sirt haqida xulosa chiqariladi.

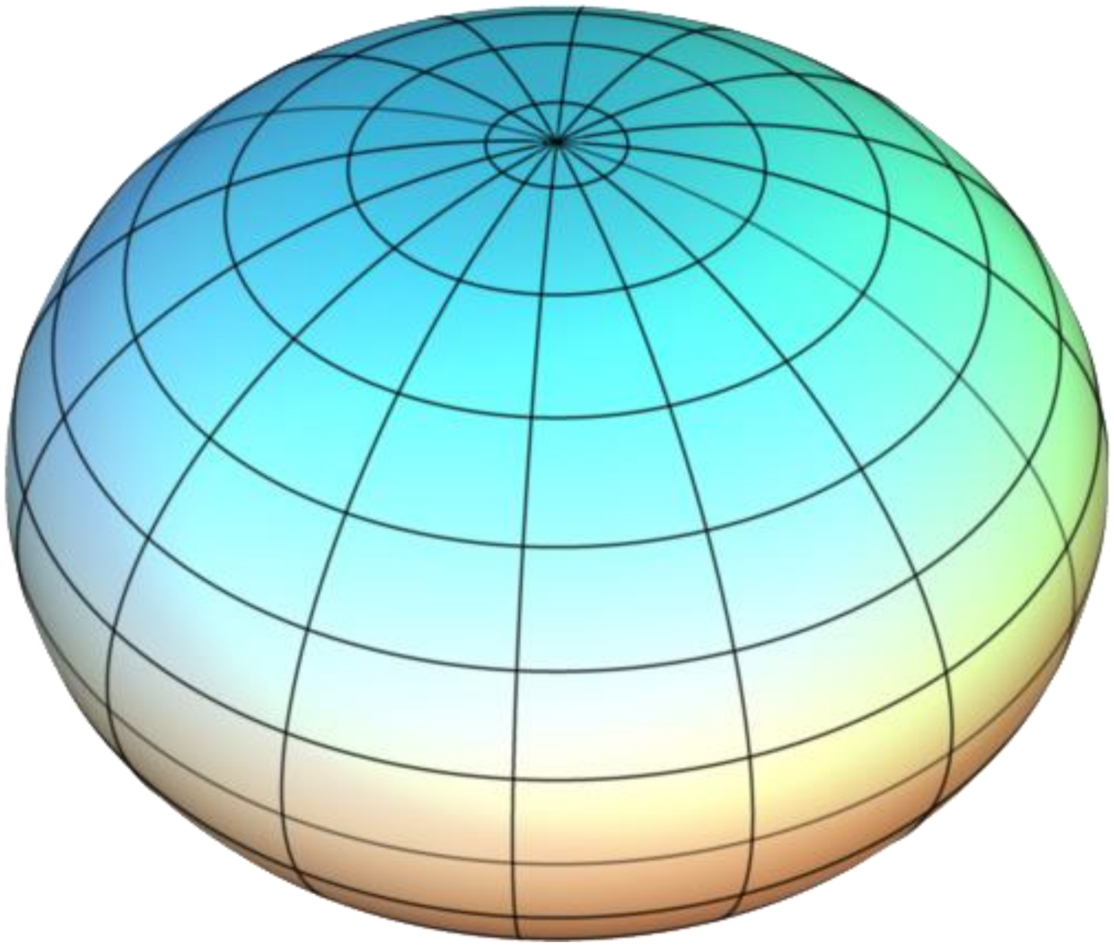
Ikkinchi tartibli sirtlarning **17 ta ko‘rinishi bor**. Sirtlarni klassifikatsiyalash g‘oyasi koordinatalar sistemasini kanonik sistemaga keltirish yo‘li bilan sirtlarning tenglamalarini kanonik ko‘rinishga keltirishga asoslangan.

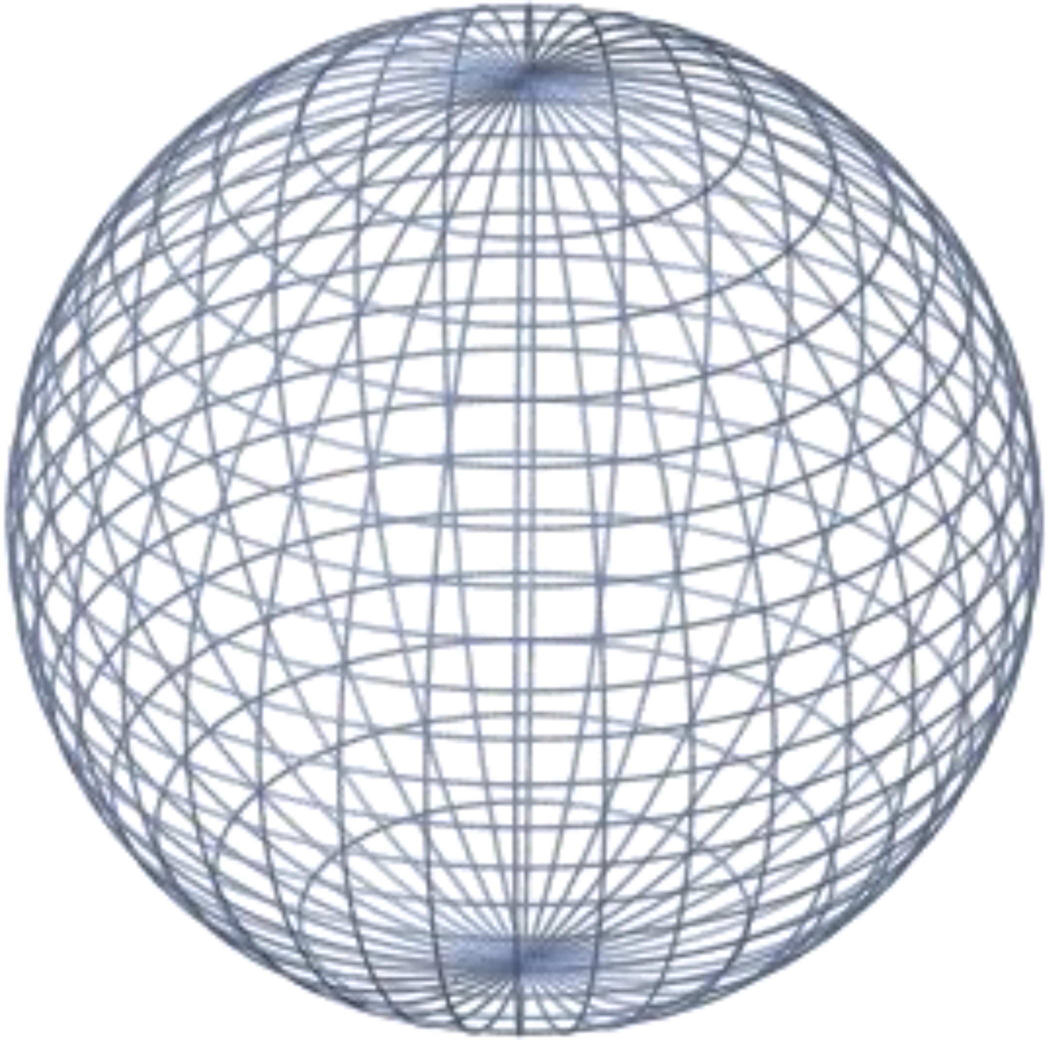
Silindrik sirtlar

Ikkinchi tartibli sirtlarning 6 ta ko‘rinishini batafsil o‘rganamiz: **ellipsoid, bir pallali giperboloid, ikki pallali giperboloid konus, elliptik paraboloid va giperbolik paraboloid**.

Ellipsoidlarning eng sodda ko‘rinishi sfera bo‘lib, bu sirt eng ko‘p o‘rganilgandir. Shuning uchun sferalarni ko‘rib chiqamiz.

Silindrik sirtlar





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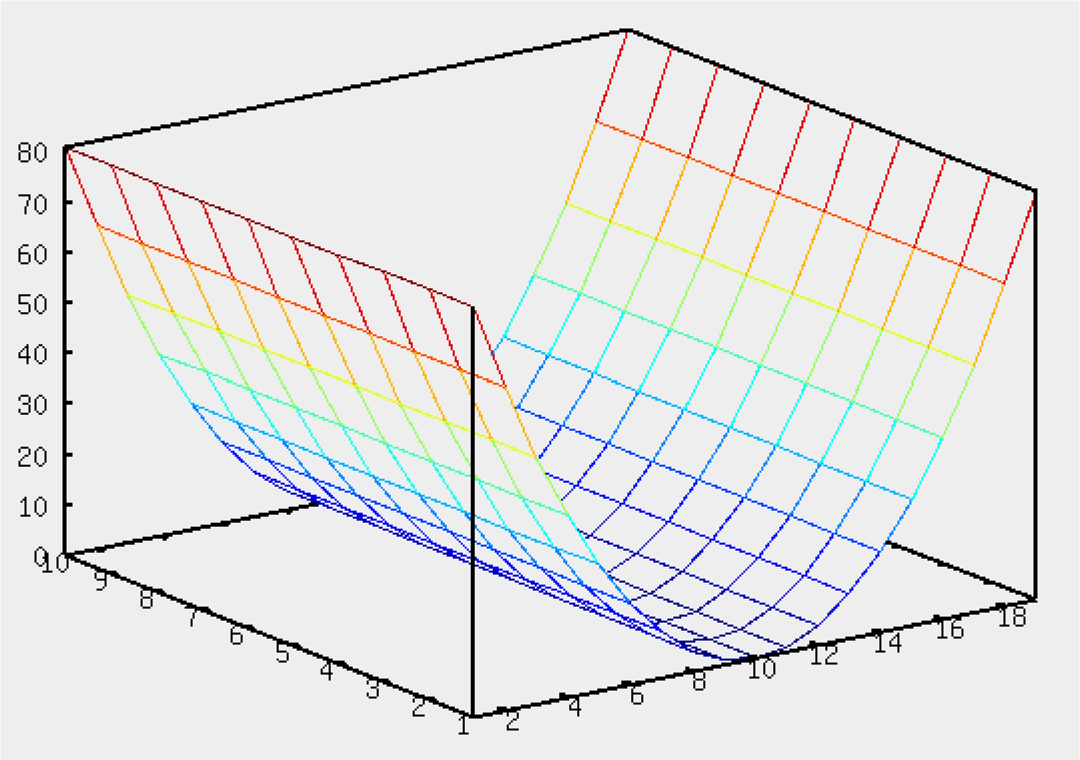

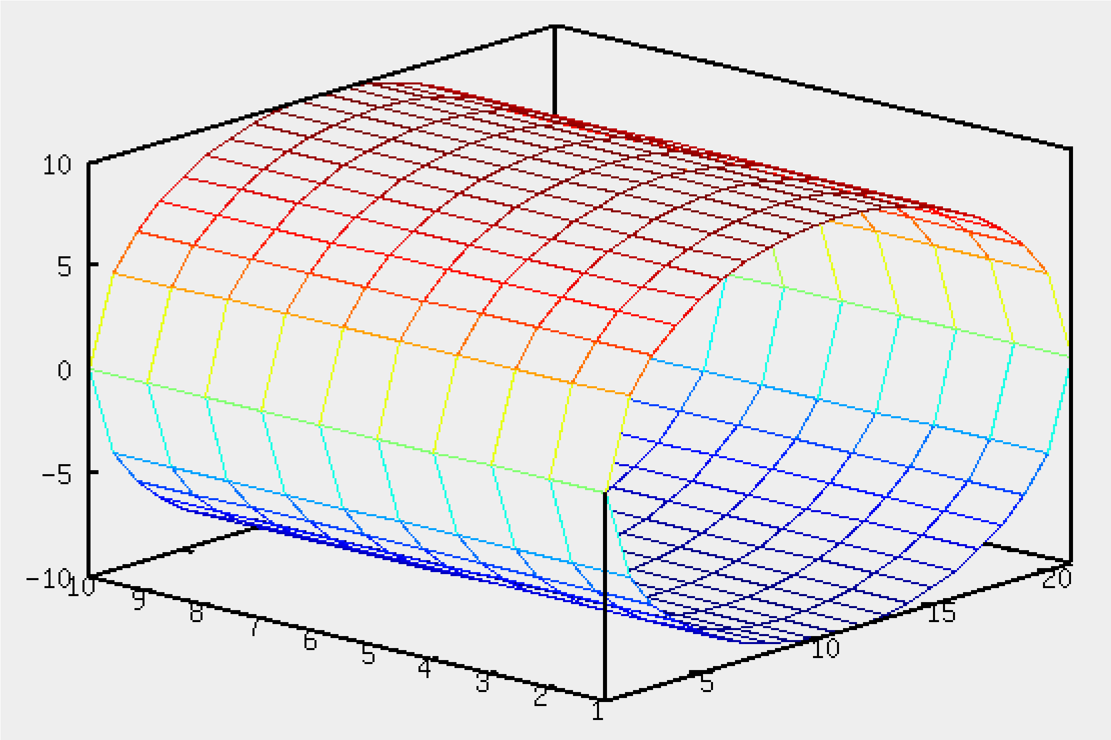

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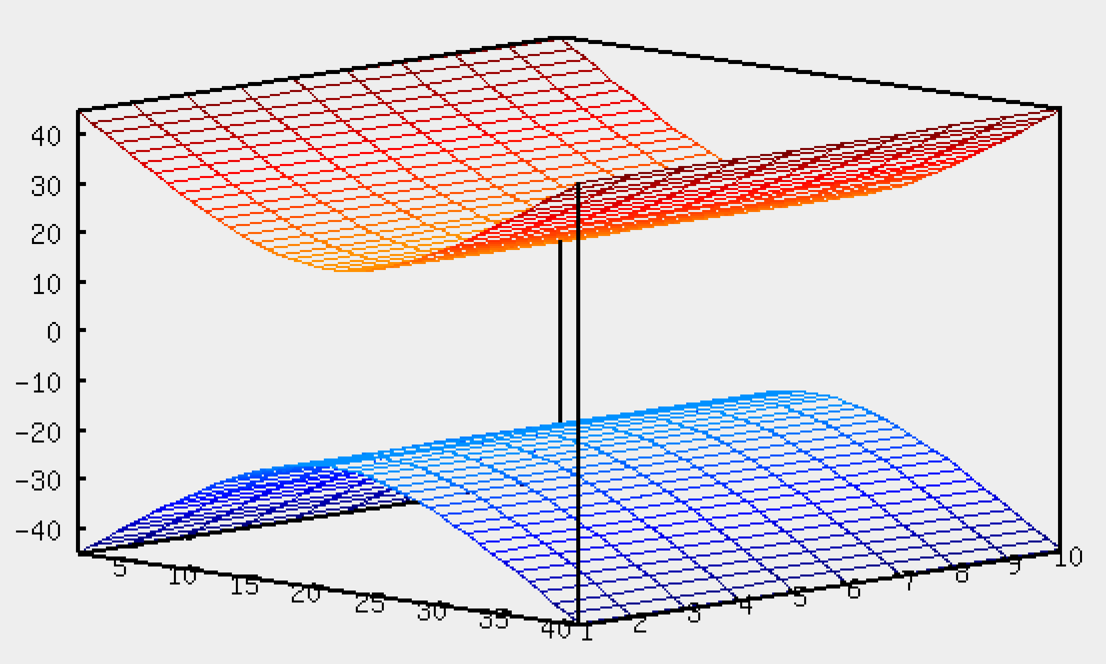

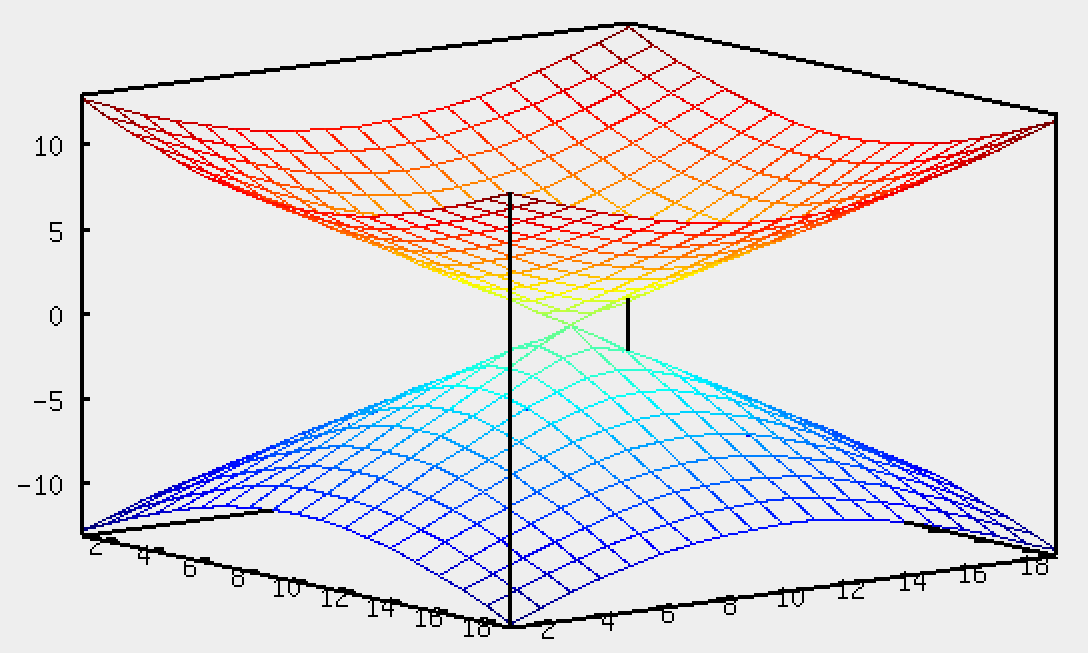

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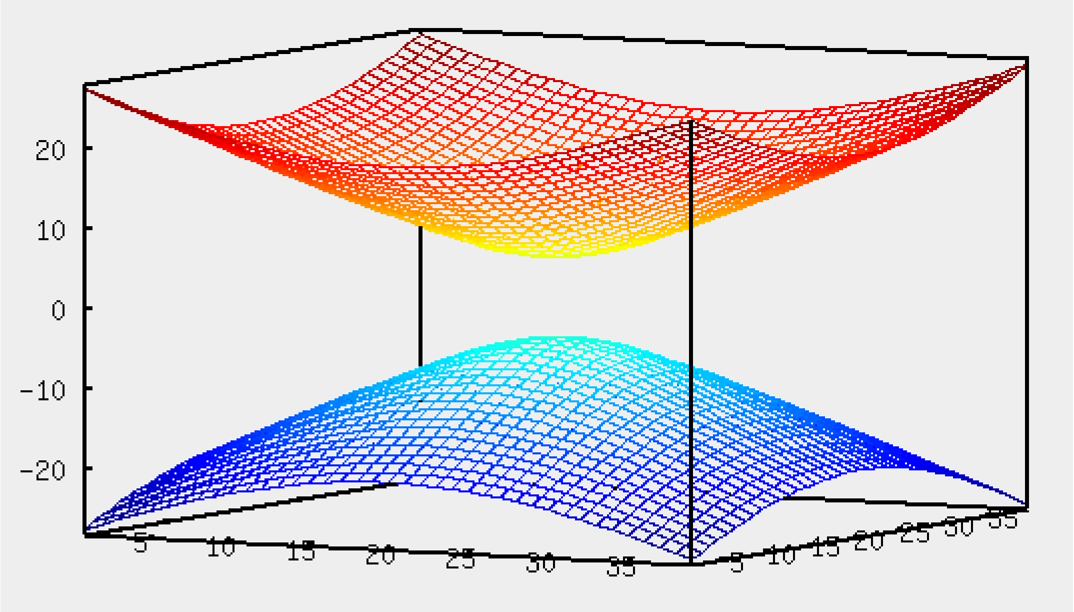
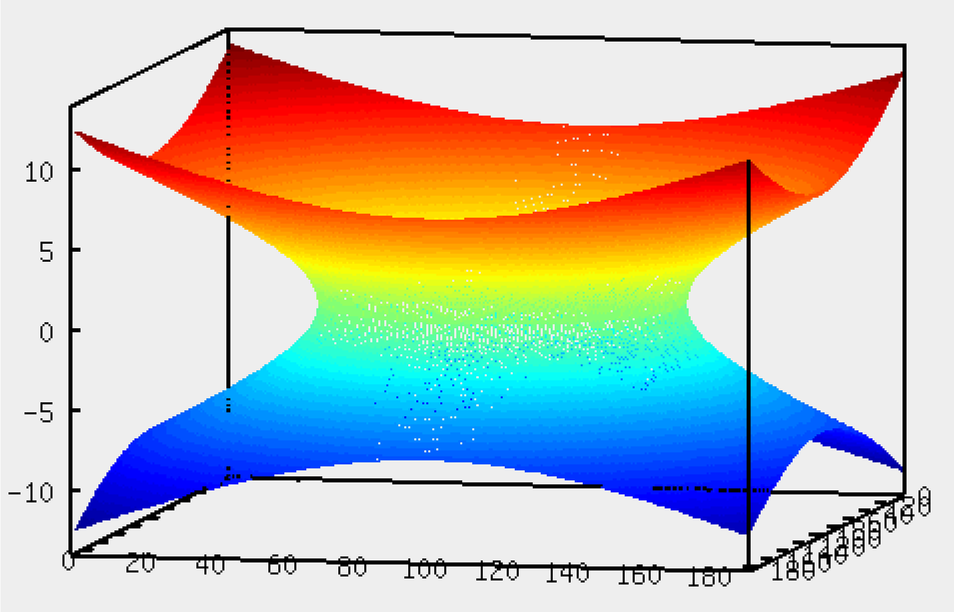
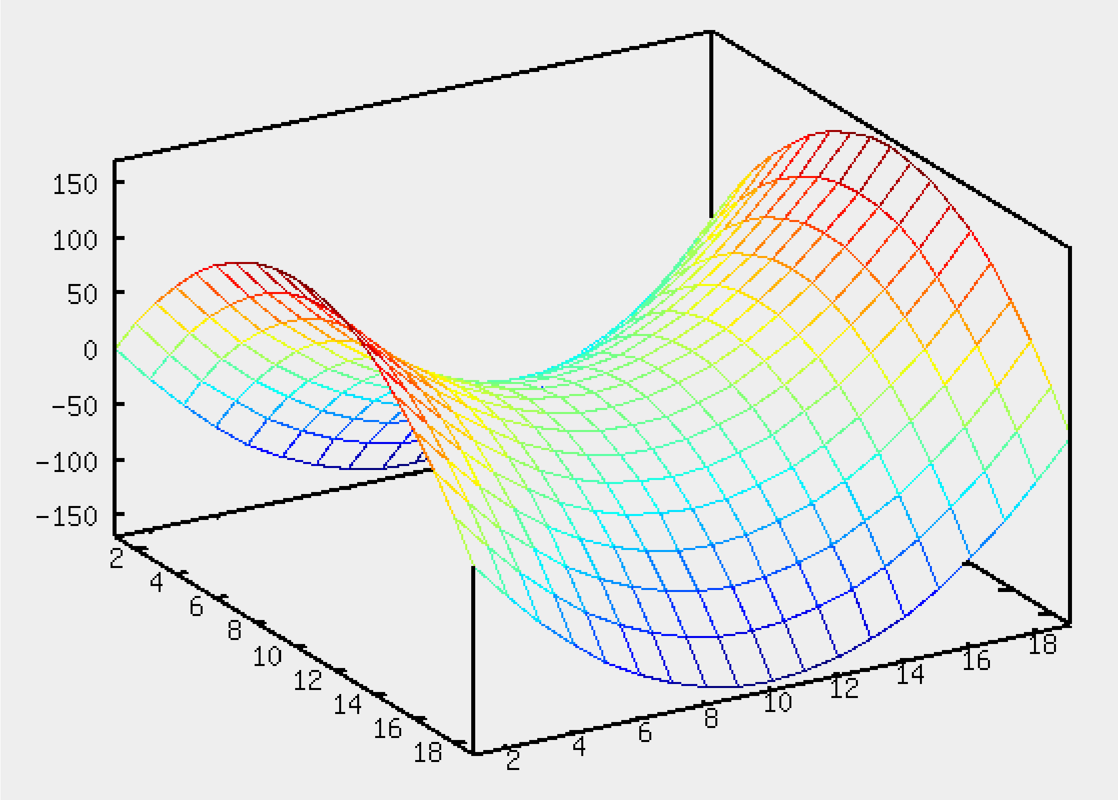
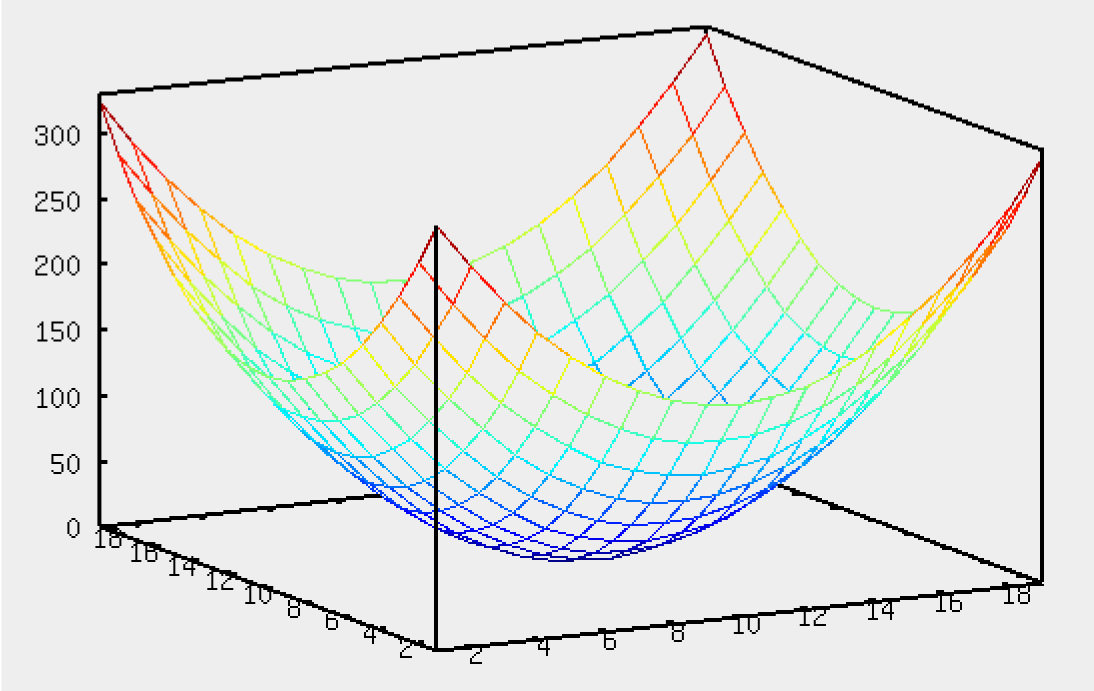




Silindrik sirtlar

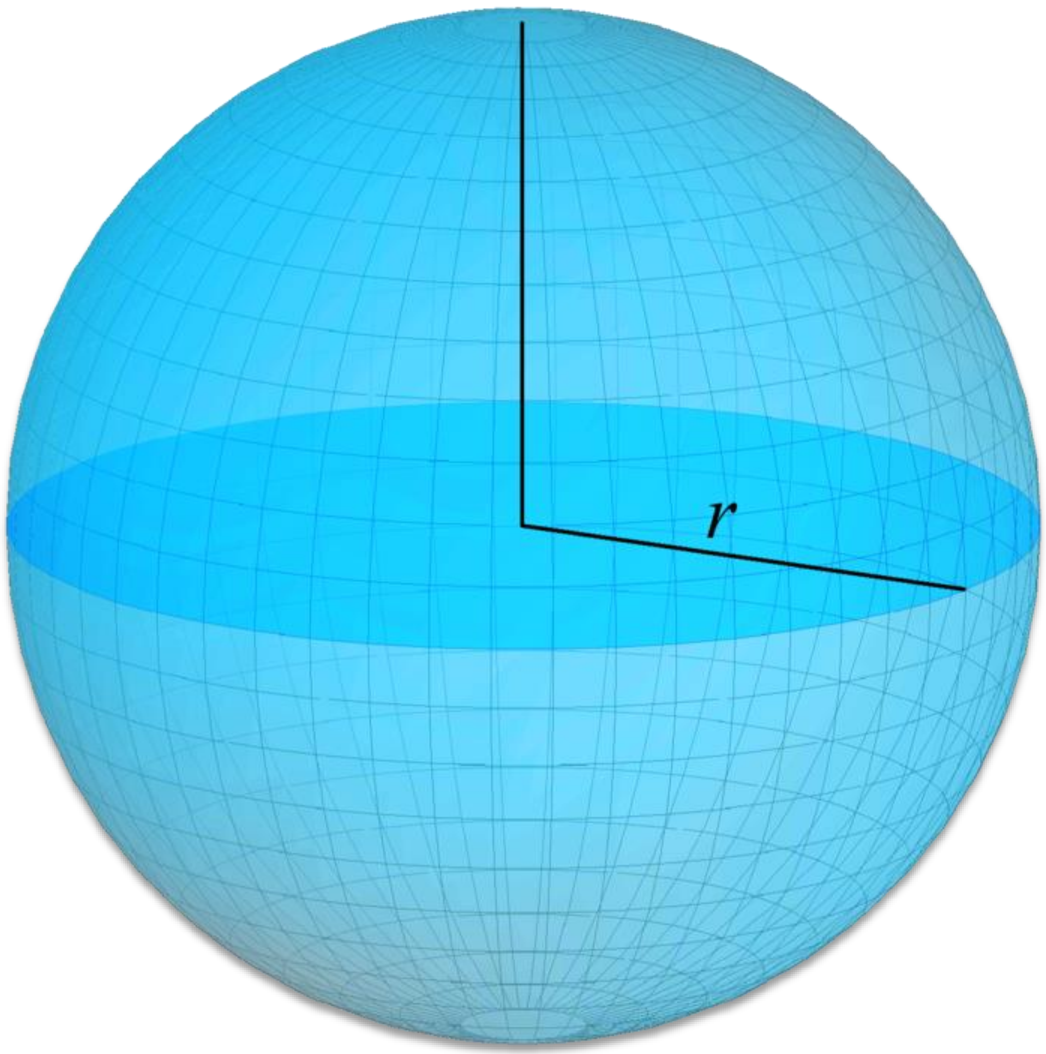




Sfera

Markazi koordinatalar boshida bo‘lgan r radiusli sfera tenglamasi

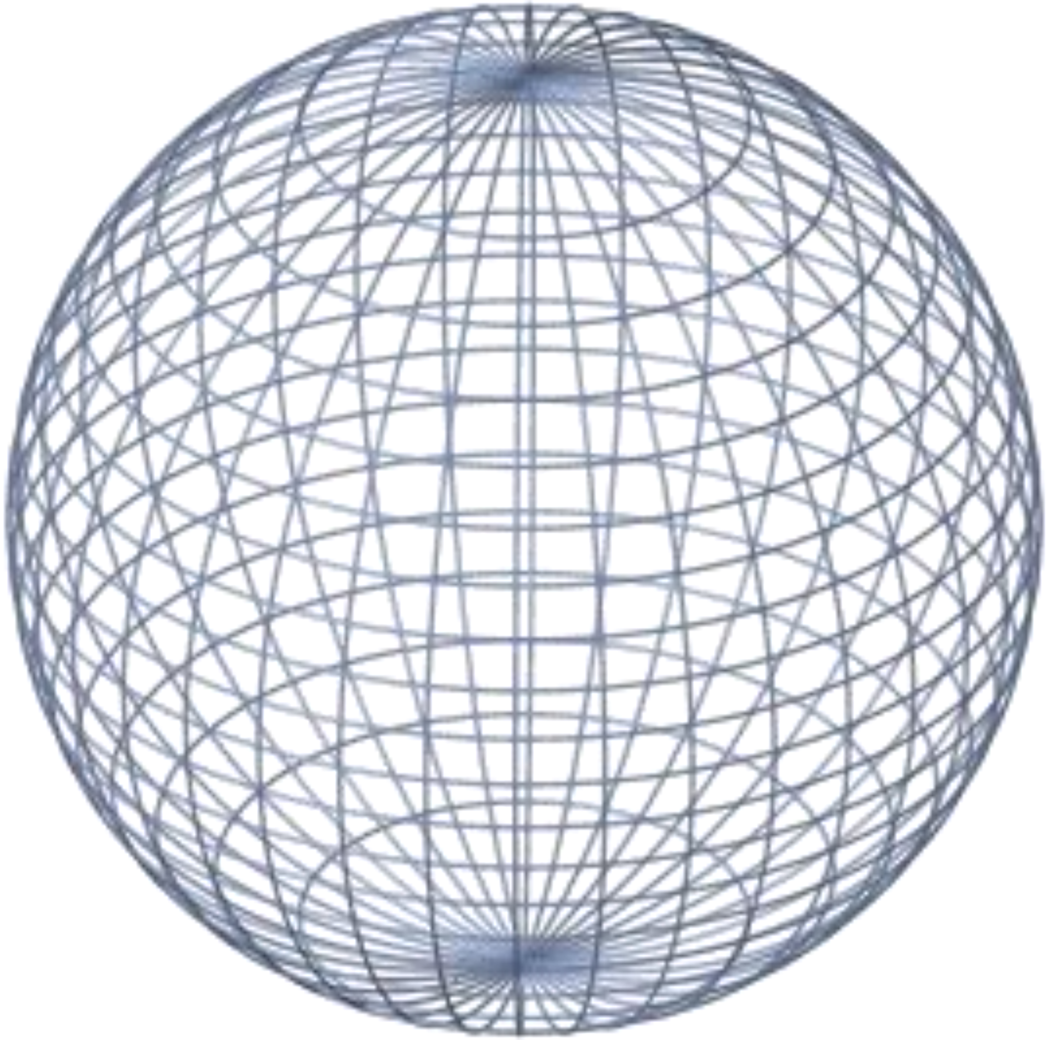
$$x^2 + y^2 + z^2 = r^2$$



Sfera

Markazi $M_0(x_0, y_0, z_0)$ nuqtada bo‘lgan R radiusli sfera tenglamasi

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$$



Ellipsoid

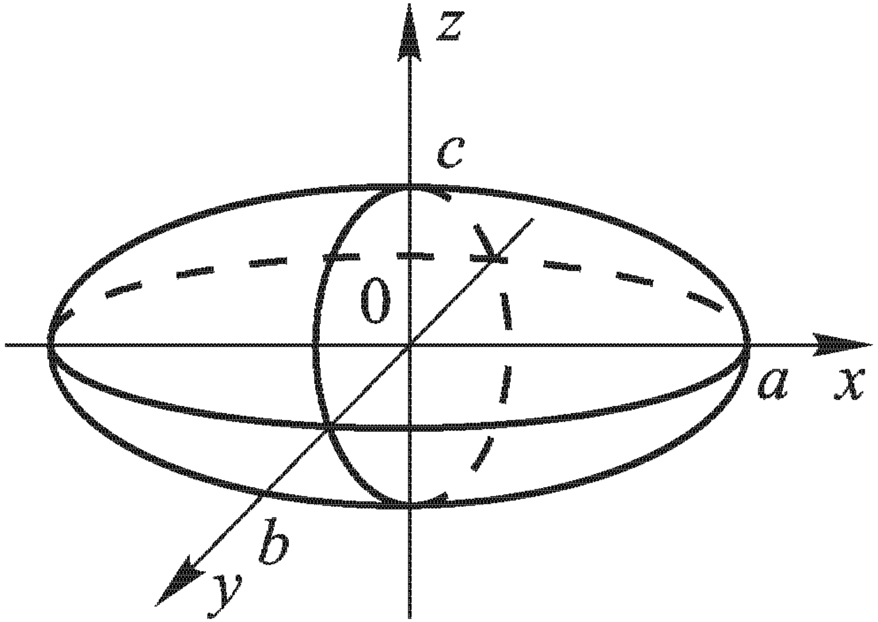
Ellipsoid deb, koordinatalarning kanonik sistemasidagi tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

ko‘rinishda bo‘lgan ikkinchi tartibli sirtga aytiladi.

Xususan, agar $a=b=c=R$ bo‘lsa, markazi koordinatalar boshida bo‘lgan R radiusli sferani olamiz.

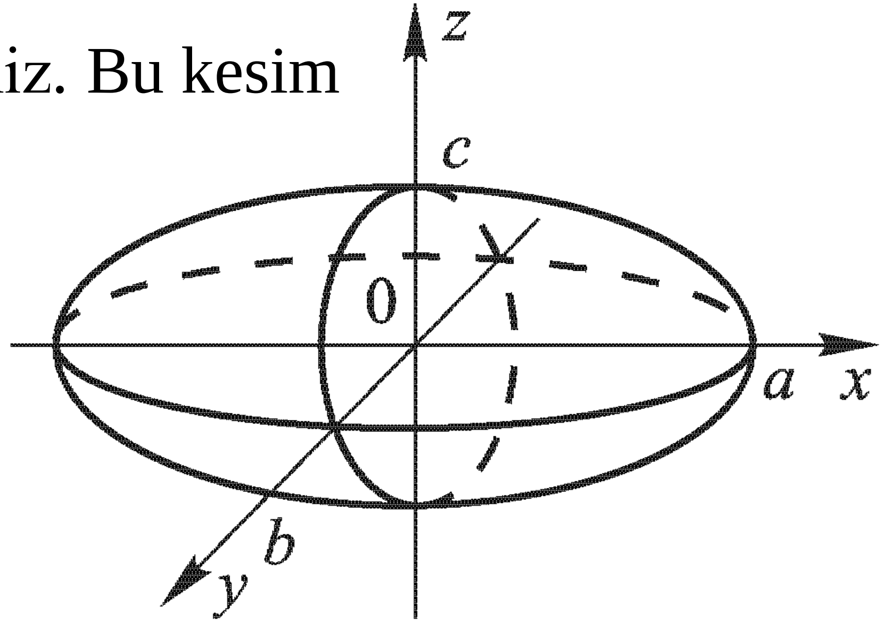
a, b, c sonlar **ellipsoidning yarim o‘qlari** deyiladi. Agar yarim o‘qlar har xil bo‘lsa, ellipsoid **uch o‘qli ellipsoid** deyiladi. Ellipsoidning koordinata o‘qlari bilan kesishgan nuqtalar **ellipsoidning uchlari** deyiladi.



Ellipsoid

Koordinatalar kanonik sistemasining o‘qlari ellipsoidning simmetriya o‘qlari, koordinatalar boshi – uning **simmetriya markazi**, koordinatalar tekisliklari esa **simmetriya tekisliklari** bo‘ladi.

Ellipsoidning $xOy: z = 0$ tekislik bilan kesimini o‘rganamiz. Bu kesim

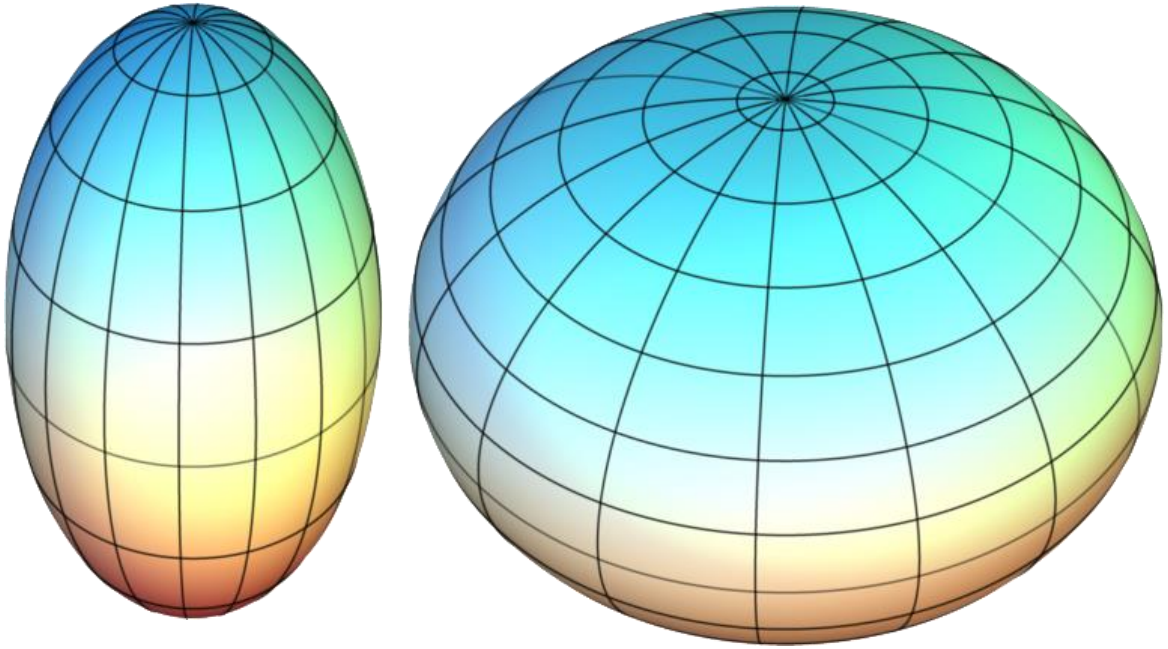


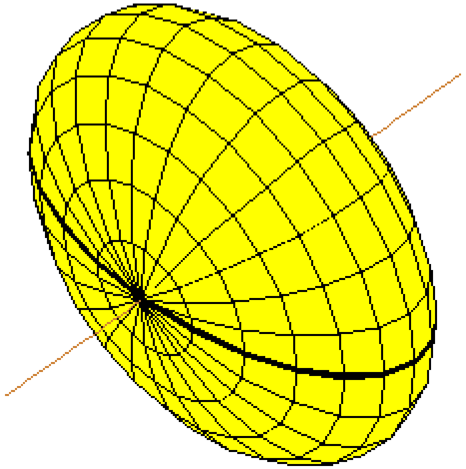
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \\ z = 0 \end{cases}$$

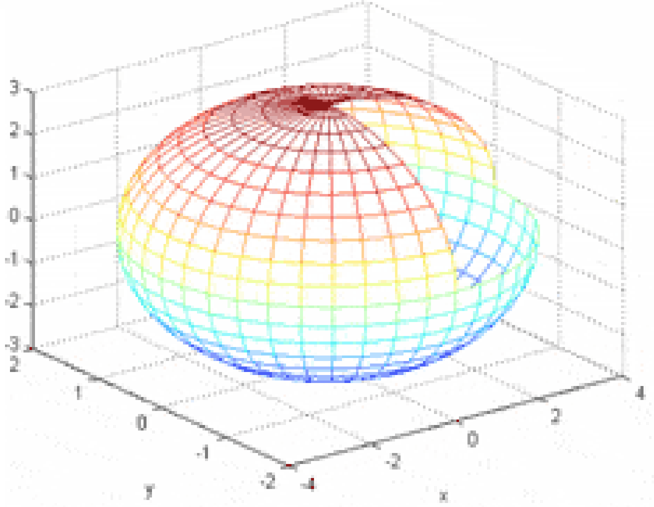
sistema bilan beriladi va kanonik tenglamasi quyidagicha bo’lgan ellipsdan iborat bo‘ladi.

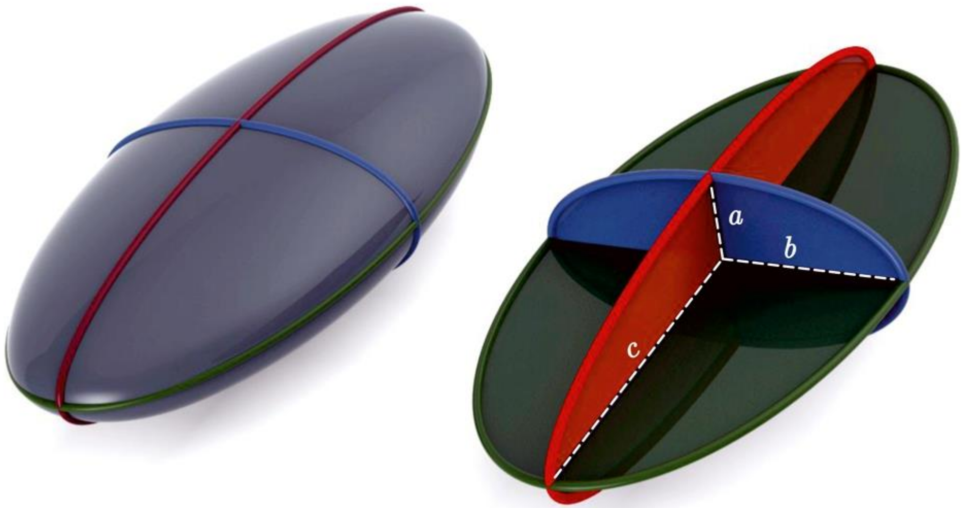
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ellipsoid





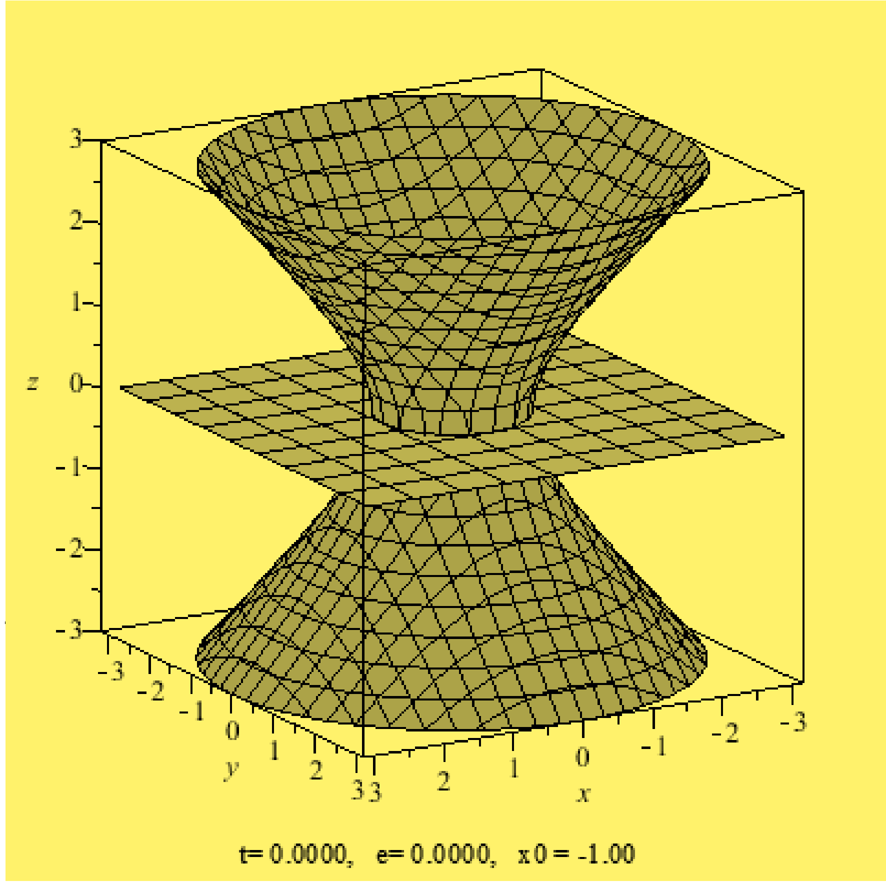




Giperboloid

Bir pallali giperboloid deb, koordinatalarning kanonik sistemasidagi quyidagi ko’rinishga ega bo’lgan sirtga aytiladi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



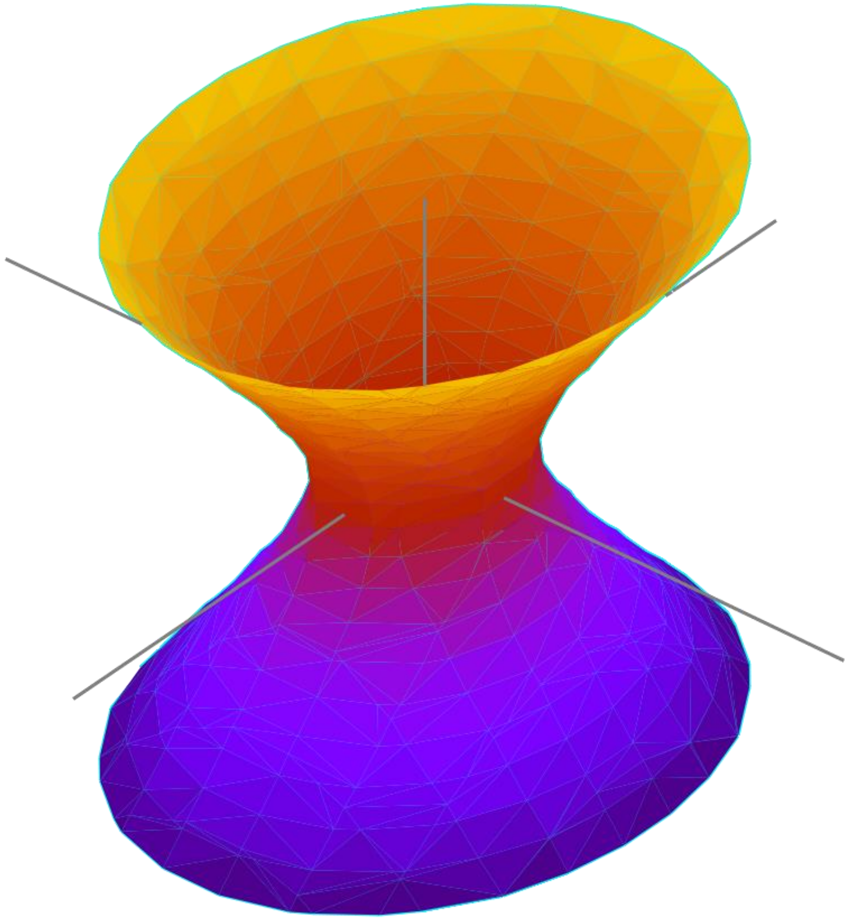
Koordinatalar kanonik sistemasining o‘qlari bir pallali giperboloidning **simmetriya o‘qlari**, koordinatalar boshi uning **simmetriya markazi**, koordinatalar tekisliklari esa **simmetriya tekisliklari** bo‘ladi.

Giperboloid

Bir pallali giperboloidning $xOz\colon y = 0$ tekislik bilan

kesimini olamiz. Bu kesim

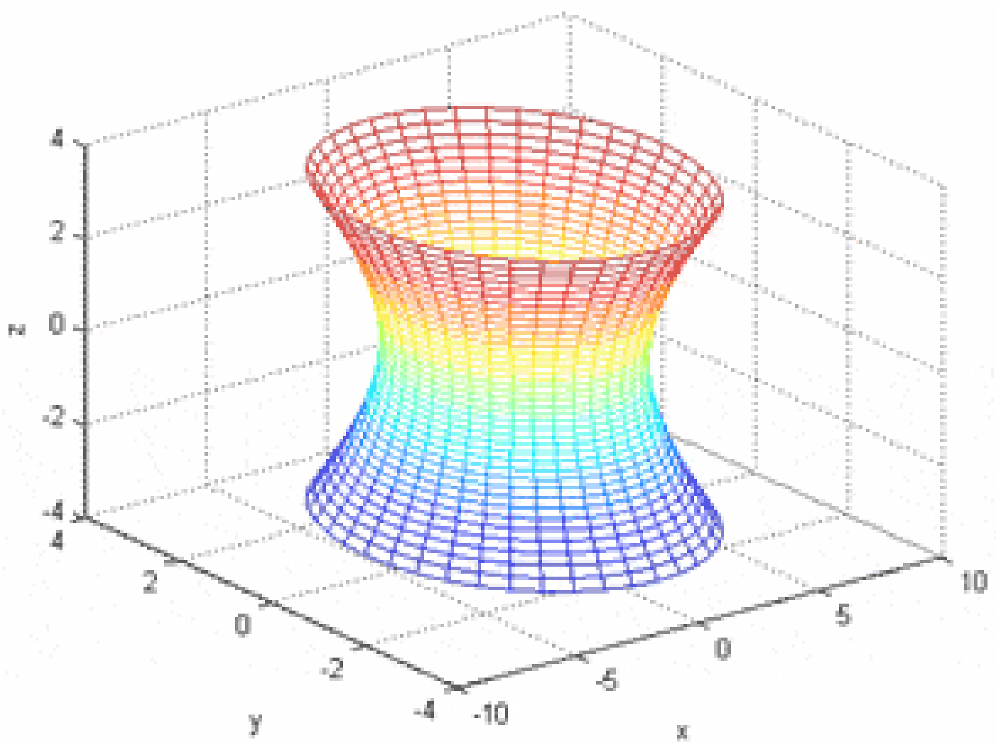
$$\begin{cases} \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1 \\ y = 0 \end{cases}$$

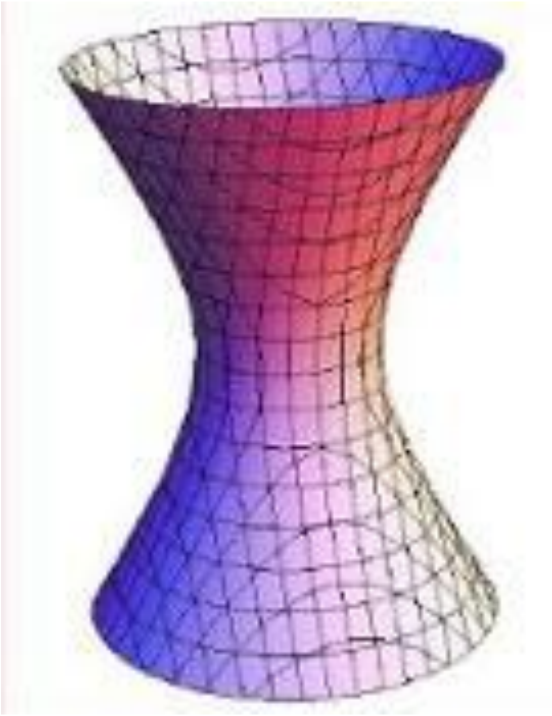


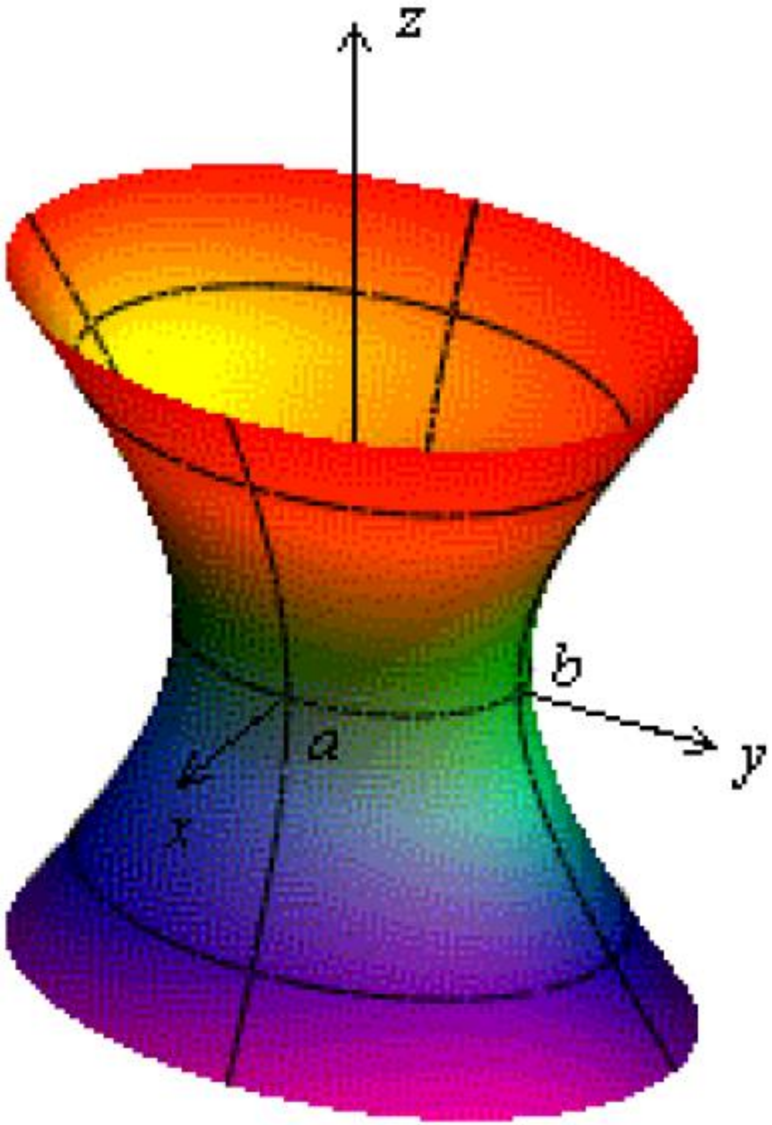
sistema bilan beriladi va kanonik tenglamasi quyidagicha bo’lgan giperboladan iborat bo‘ladi.

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$$

Bir pallali giperboloidlar



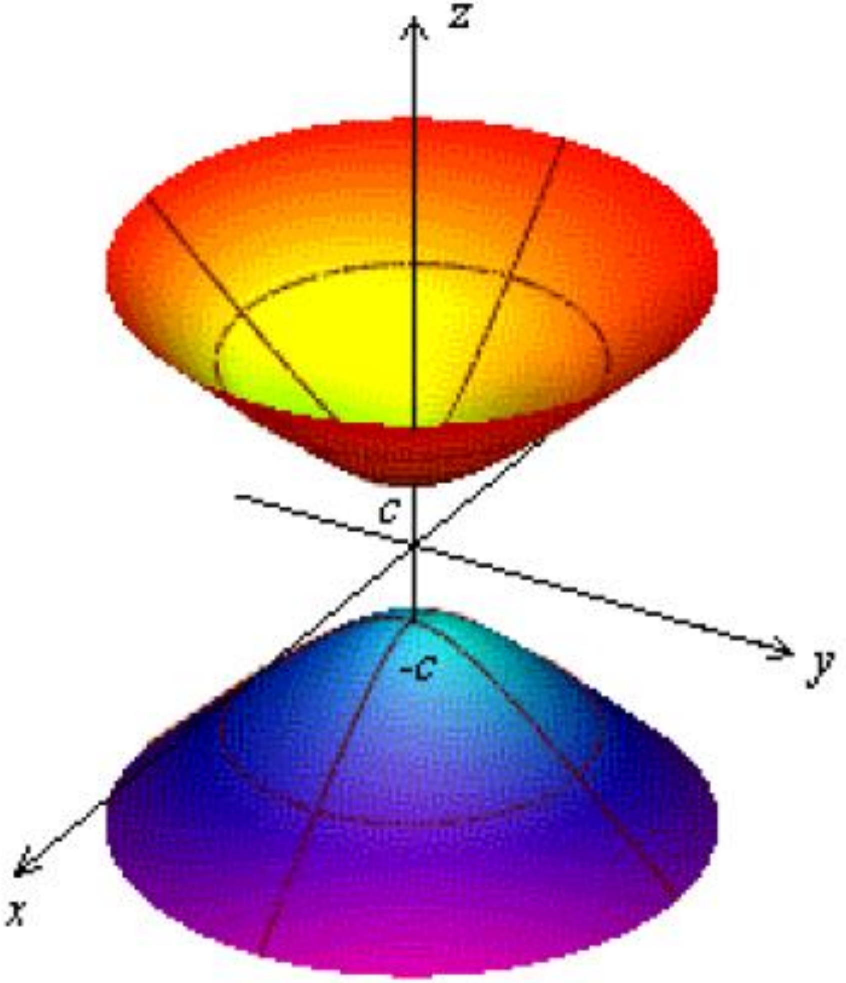




Giperboloid

Ikki pallali giperboloid deb, koordinatalarning kanonik sistemasidagi quyidagi ko'rinishga ega bo'lgan sirtga aytiladi

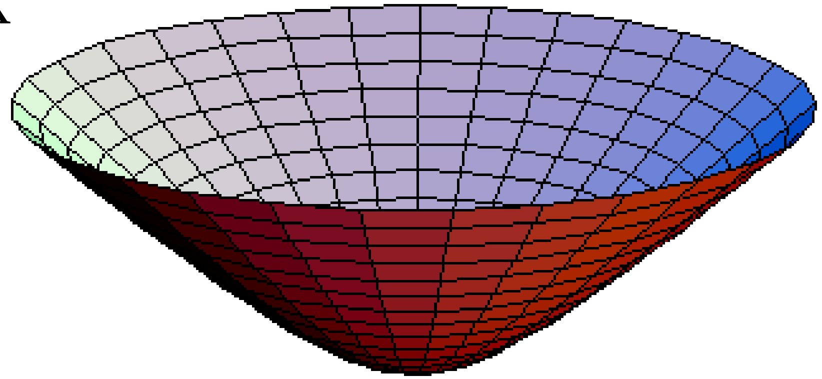
$$\frac{x^2}{a^2}+\frac{y^2}{b}-\frac{z^2}{c^2}=-1$$



Koordinatalar kanonik sistemasining o'qlari bir pallali giperboloidning **simmetriya o'qlari**, koordinatalar boshi – uning **simmetriya markazi**, koordinatalar tekisliklari esa **simmetriya tekisliklari** bo'ladi.

Giperboloid

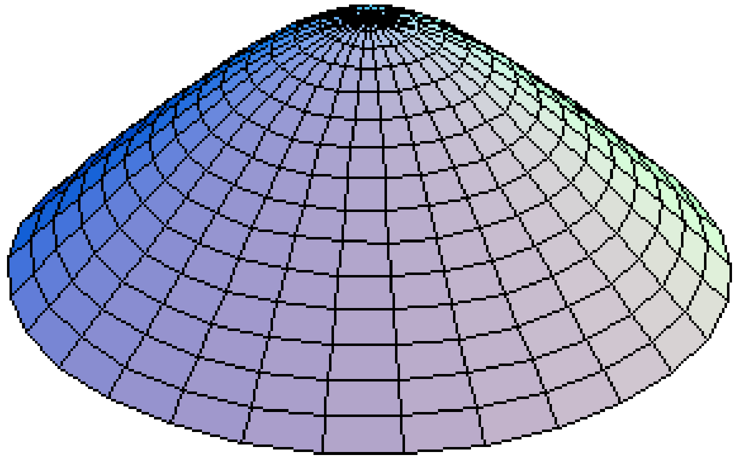
Bir pallali giperboloidning $xOz: y = 0$ yoki $z = h$ tekislik



bilan kesimini olamiz. Bu kesim

$$\begin{cases} \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = -1 \\ z = h \end{cases}$$

sistema bilan beriladi va kanonik tenglamasi quyidagicha bo'lgan ellipsdan iborat bo'ladi.

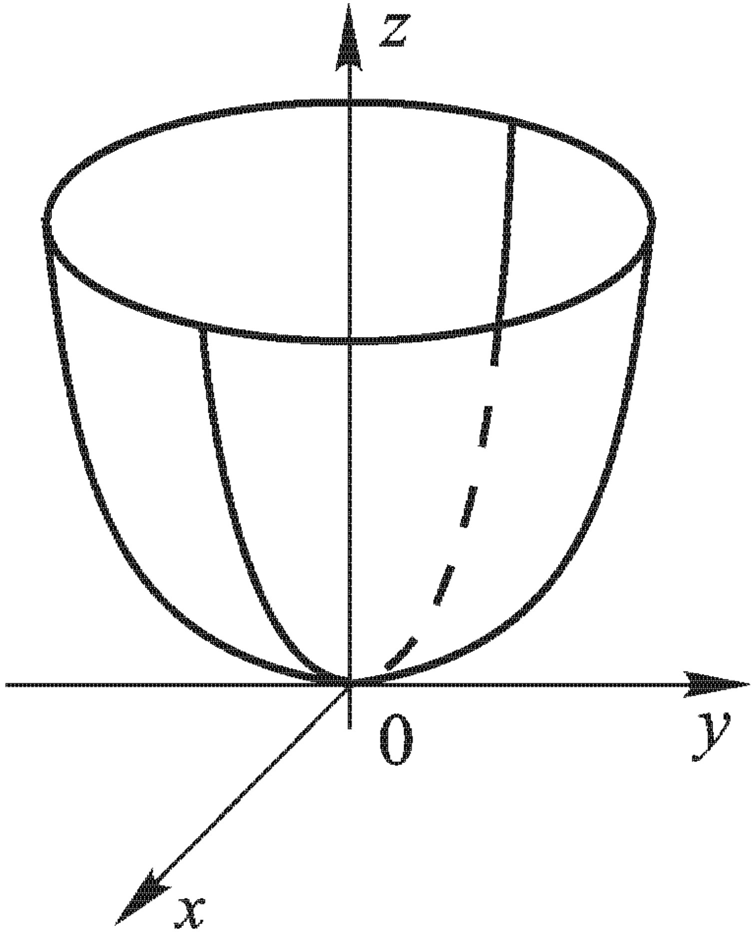


$$\frac{x^2}{\left(\frac{h^2}{c^2} - 1\right)a^2} + \frac{y^2}{\left(\frac{h^2}{c^2} - 1\right)b^2} = 1$$

Paraboloid

Elliptik paraboloid deb, koordinatalarning kanonik sistemasidagi quyidagi ko'rinishga ega bo'lgan sirtga aytiladi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$$



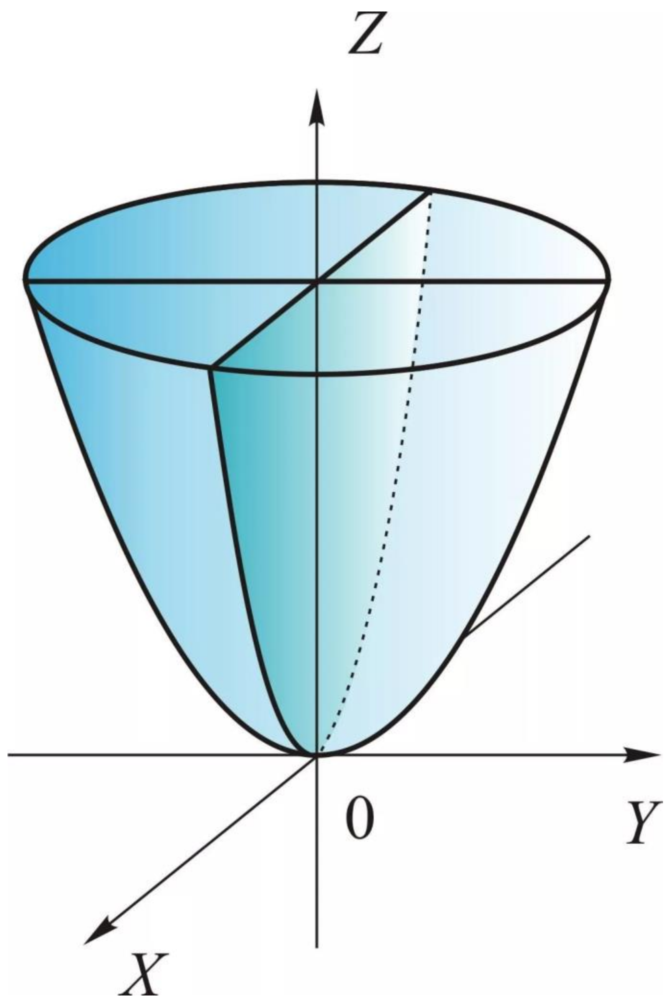
Paraboloid

Elliptik paraboloidning $z = h$ gorizontal tekisliklar bilan kesimida ellipslar hosil bo‘ladi:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2h$$

Elliptik paraboloidning $x = h$ va $y = h$ vertikal tekisliklar bilan kesimida parabolalar hosil bo‘ladi:

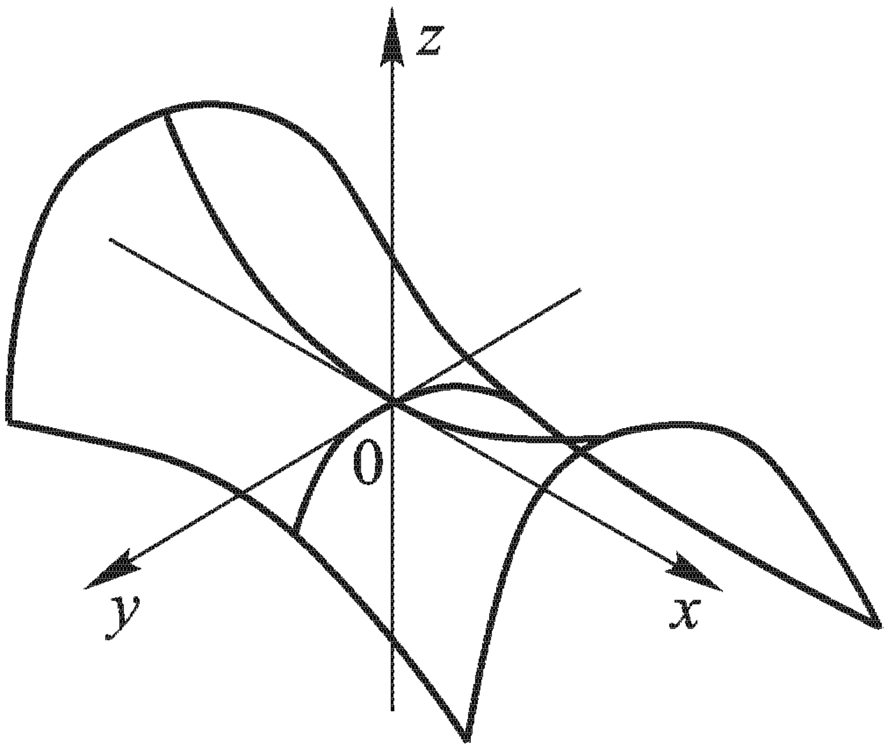
$$\frac{y^2}{b^2} = 2z - \frac{h^2}{a^2} \quad \text{yoki} \quad \frac{x^2}{a^2} = 2z - \frac{h^2}{b^2}.$$



Paraboloid

Giperbolik paraboloid deb, koordinatalarning kanonik sistemasidagi quyidagi ko'rinishga ega bo'lgan sirtga aytiladi

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$$

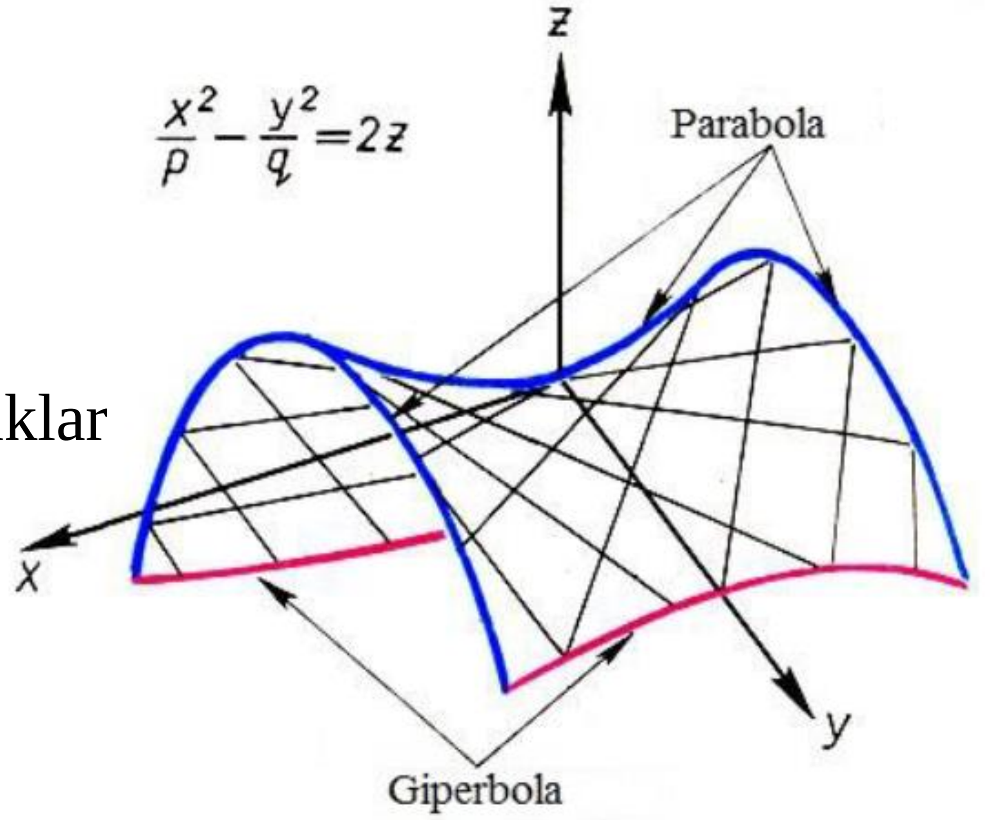


Paraboloid

Giperbolik paraboloidning $z = h$ gorizontal tekisliklar

bilan kesimida giperbolalar hosil bo‘ladi:

$$\frac{x^2}{2a^2h} - \frac{y^2}{2b^2h} = 1$$



Elliptik paraboloidning $x = h$ va $y = h$ vertikal tekisliklar

bilan kesimida parabolalar hosil bo‘ladi:

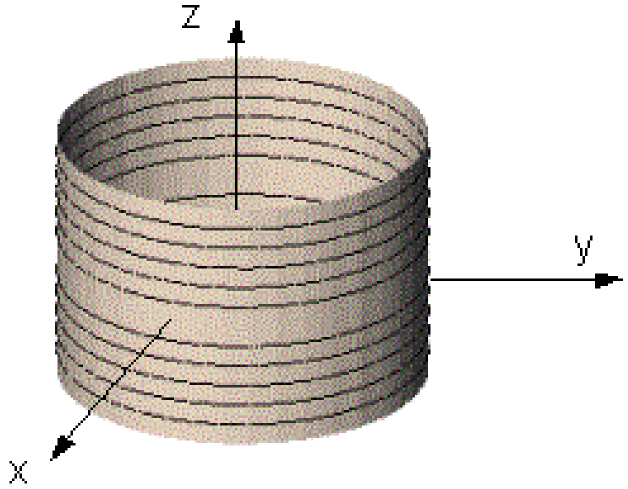
$$\frac{y^2}{b^2} = -2z + \frac{h^2}{a^2} \quad \text{va} \quad \frac{x^2}{a^2} = 2z + \frac{h^2}{b^2}.$$

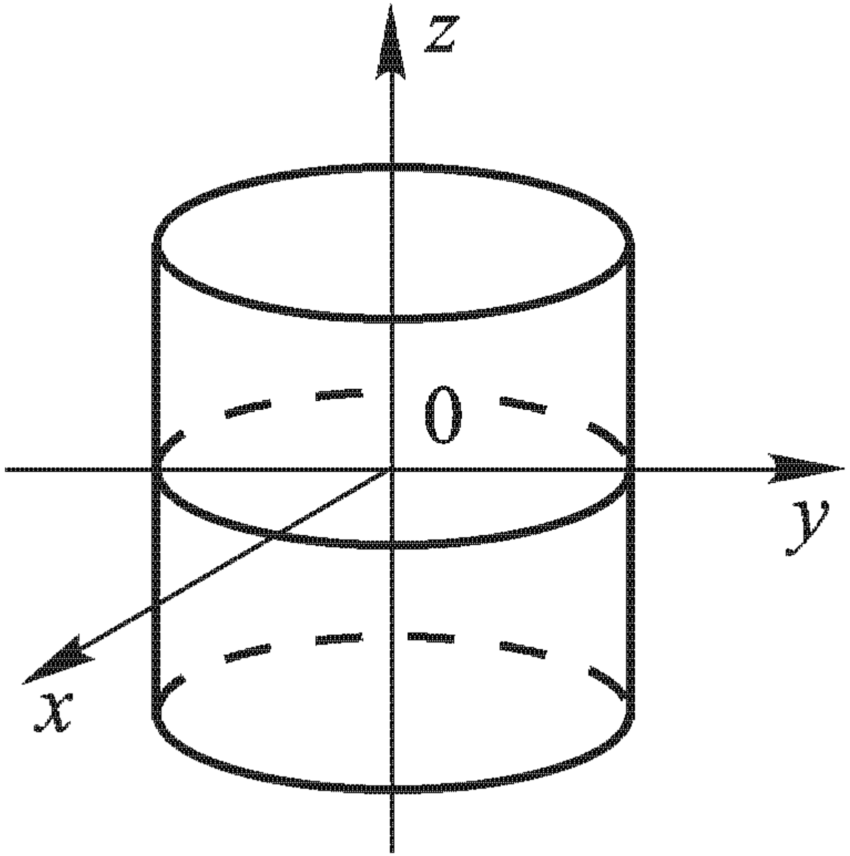
Silindrik sirtlar

Elliptik silindr deb, koordinatalarning kanonik sistemasidagi quyidagi ko’rinishga ega bo’lgan sirtga aytiladi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Agar a=b=R bo‘lsa, $x^2+y^2=R^2$ doiraviy silindr hosil bo‘ladi.

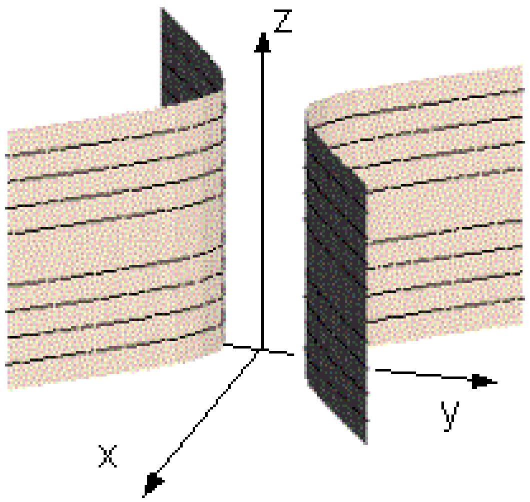


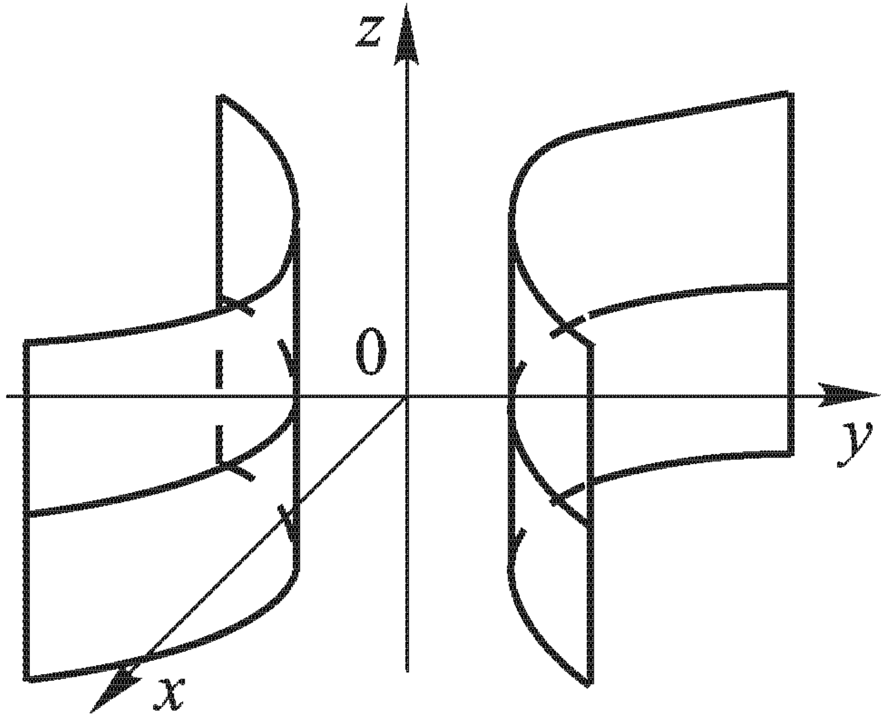


Silindrik sirtlar

Giperbolik silindr deb, koordinatalarning kanonik sistemasidagi quyidagi ko'rinishga ega bo'lgan sirtga aytiladi

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

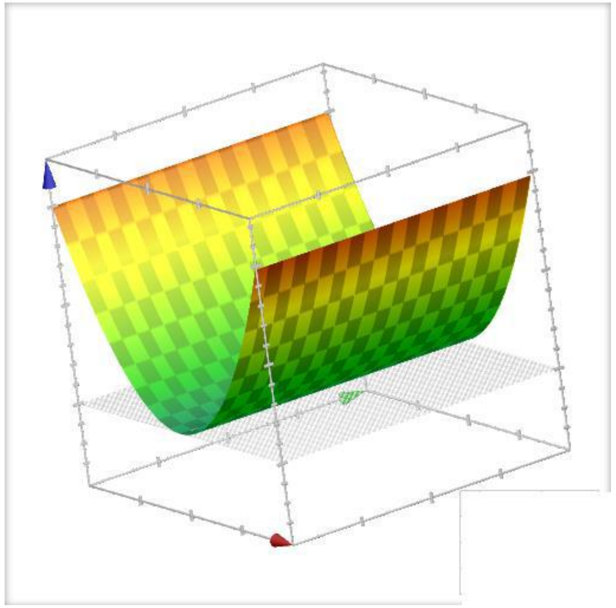


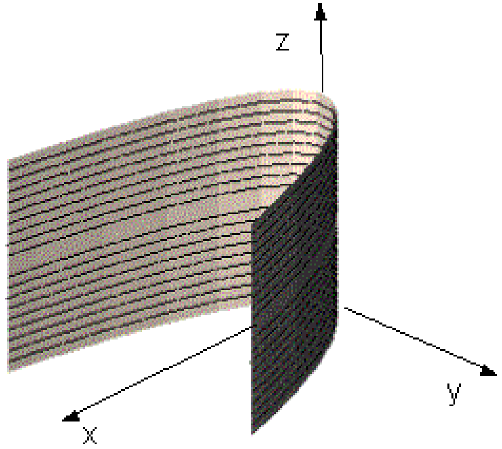


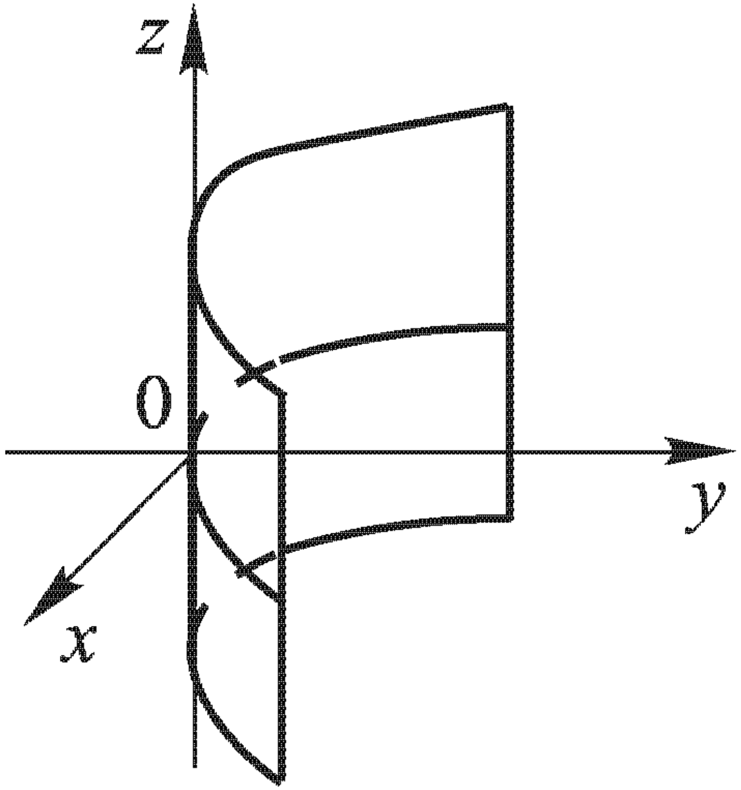
Silindrik sirtlar

Parabolik silindr deb, koordinatalarning kanonik sistemasidagi quyidagi ko'rinishga ega bo'lgan sirtga aytiladi

$$x^2 = 2py$$







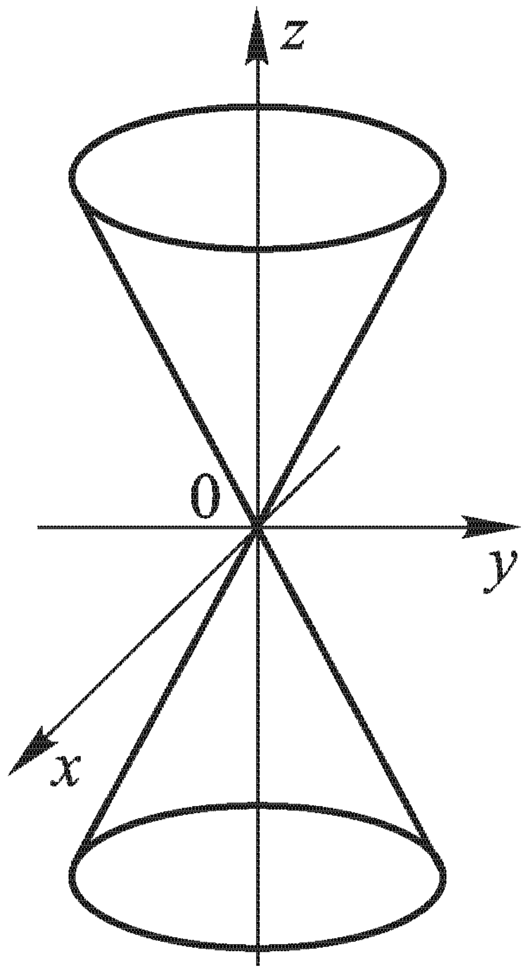
Ikkinchi tartibli konus

Elliptik konus deb, koordinatalarning kanonik sistemasidagi quyidagi ko’rinishga ega bo’lgan sirtga aytiladi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$$

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$$



Ikkinchi tartibli konus

Agar $a = b$ bo‘lsa, u holda doiraviy konus hosil bo‘ladi.

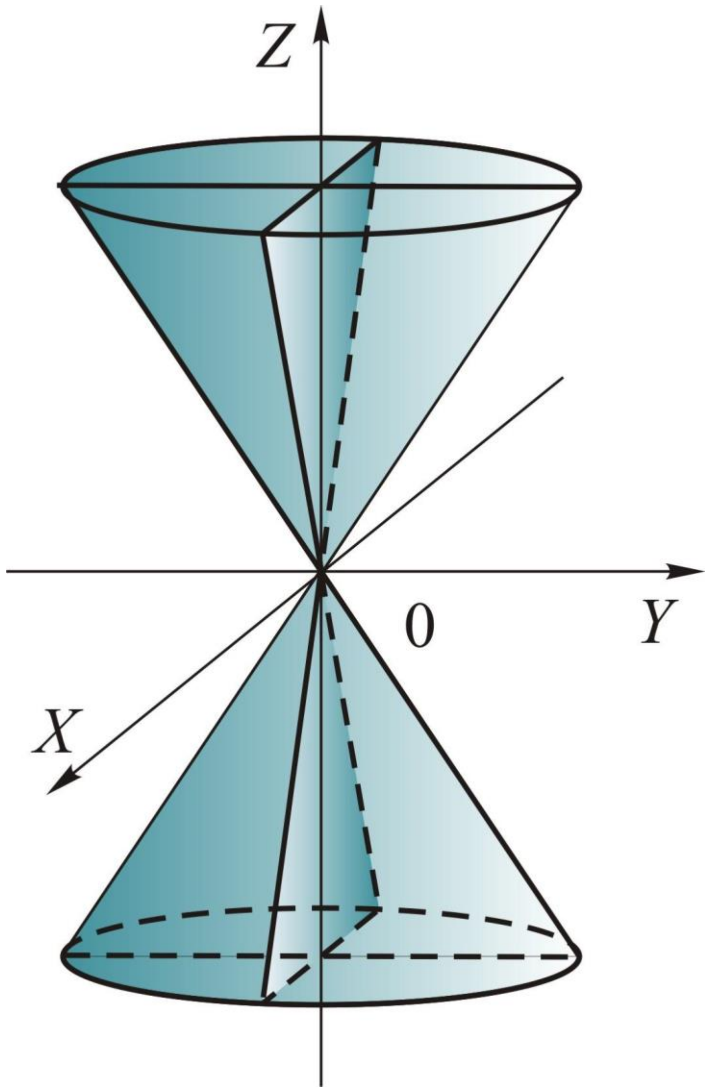
Konusning $z = h$ gorizontal tekisliklar bilan kesimida ellipslar hosil bo‘ladi ($h = 0$ bo‘lganda konus nuqtaga aylanib qoladi):

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$$

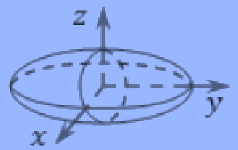
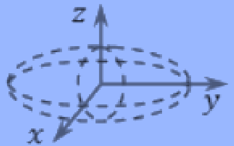
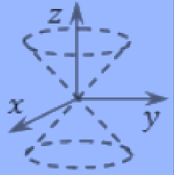
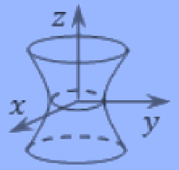
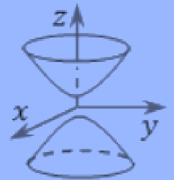
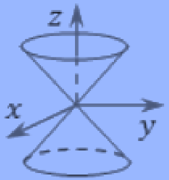
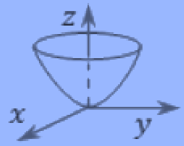
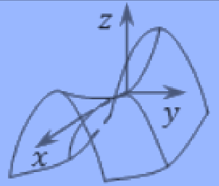
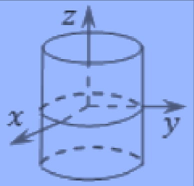
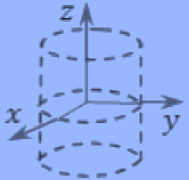
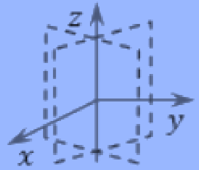
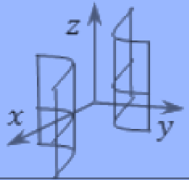
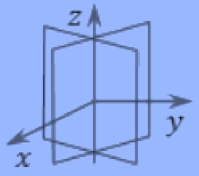
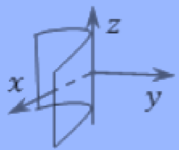
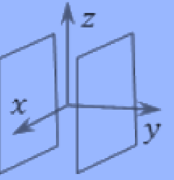
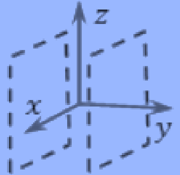
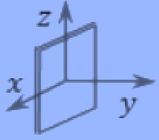
Konusningning $x = h$ va $y = h$ vertikal tekisliklar bilan kesimida giperbolalar hosil bo‘ladi:

$$\frac{y^2}{b^2}-\frac{z^2}{c^2}=-\frac{h^2}{a^2}$$

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=-\frac{h^2}{b^2}$$



Ikkinchi tartibli sirtlar

1.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Уравнение эллипсоида 	2.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$ Уравнение мнимого эллипсоида 	3.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$ Уравнение мнимого конуса 
4.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Уравнение однополостного гиперболоида 	5.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$ Уравнение двуполостного гиперболоида 	6.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$ Уравнение конуса 
7.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ Уравнение эллиптического параболоида 	8.	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$ Уравнение гиперболического параболоида 	9.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Уравнение эллиптического цилиндра 
10.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=-1$ Уравнение мнимого эллиптического цилиндра 	11.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=0$ Уравнение пары мнимых пересекающихся плоскостей 	12.	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ Уравнение гиперболического цилиндра 
13.	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$ Уравнение пары пересекающихся плоскостей 	14.	$y^2=2px$ Уравнение параболического цилиндра 	15.	$y^2-b^2=0$ Уравнение пары параллельных плоскостей 
16.	$y^2+b^2=0$ Уравнение пары мнимых параллельных плоскостей 	17.	$y^2=0$ Уравнение пары совпадающих плоскостей 		Для всех уравнений $a>0,\ b>0,\ c>0,\ p>0$ Для уравнений 1 и 2 $a\geq b\geq c$ Для уравнений 3,4,5,6,7,9,10 $a\geq b$

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